1.4 Rational Exponents

A. Use a calculator to complete the tables.

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$

squared

perfect squares

x	$x^{\frac{1}{3}}$
1	$1^{1/3} = 1$
8	$8^{1/3} = 2$
27	$27^{1/3} = 3$
64	$64^{1/3} = 4$

cubed

perfect cubes

Notice the pattern:

$\sqrt{x}$ **the** square root **as a power is** $x^{1/2}$

$\sqrt[3]{x}$ **the** cube root **as a power is** $x^{1/3}$

$\sqrt[5]{x}$ **the** fifth root **as a power is** $x^{1/5}$

> IN GENERAL $x^{\frac{1}{n}}$ (power) as a radical becomes $\sqrt[n]{x}$ (radical). And vice versa, $\sqrt[n]{x}$ equals $x^{\frac{1}{n}}$.

Example 1: Write as a radical and then evaluate.

a) $1000^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= 10$

b) $0.25^{\frac{1}{2}}$

$= \sqrt[2]{0.25}$

$= 0.5$

c) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$= \sqrt[4]{\frac{16}{81}}$

$= \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$

> $x^{\frac{m}{n}}$ ← m: *Exponent of the root or radicand*; ← n: *Index of the root (little #)*
>
> so $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $\left(\sqrt[n]{x}\right)^m$

base, index, radicand, stays exponent

← easier to evaluate the root first, then the exponent.

Example 2: Write as a power.

a) $\sqrt[2]{3^5}$

$= 3^{\frac{5}{2}}$

b) $\left(\sqrt[3]{25}\right)^2$

$= 25^{\frac{2}{3}}$

Example 3: Write as a radical and then evaluate.

a) $8^{\frac{2}{3}} = \sqrt[3]{8^2}$

$= (2)^2$

$= 4$

b) $(-27)^{\frac{4}{3}} = \sqrt[3]{(-27)^4}$

$= (-3)^4$

$= 81$

decimal → fraction

c) $(-32)^{0.4}$, $0.4 = \frac{4}{10} = \frac{2}{5}$

$(-32)^{\frac{2}{5}} = \sqrt[5]{(-32)^2}$

$= (-2)^2$

$= 4$

flip base

d) $8^{-\frac{2}{3}}$ ← radical

$= \sqrt[3]{\left(\frac{1}{8}\right)}^{\,2}$

$= \left(\frac{\sqrt[3]{1}}{\sqrt[3]{8}}\right)^2$

$= \left(\frac{1}{2}\right)^2$

$= \frac{1}{4}$

e) $(-0.027)^{-\frac{2}{3}}$, $\frac{27}{1000}$

$= \sqrt[3]{\left(-\frac{1000}{27}\right)}^{\,2}$

$= \left(\frac{\sqrt[3]{(-1000)}}{\sqrt[3]{27}}\right)^2$

$= \left(\frac{-10}{3}\right)^2 = \frac{100}{9}$

HW: 1.4 WS

Pre-Calc 11