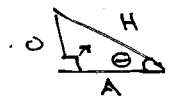
2.1 Angles in Standard Position

What is Trigonometry? A branch of Math that analyses and calculates the relations between the angles and the side lengths of triangles.



*calc. must be in degree mode.

We know from grade 10 math: Sin A = $\frac{O}{H}$ Cos A = $\frac{A}{H}$ Tan A = $\frac{O}{A}$

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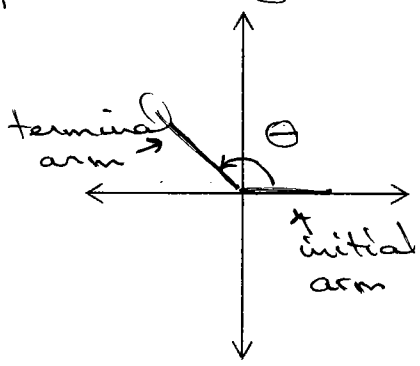
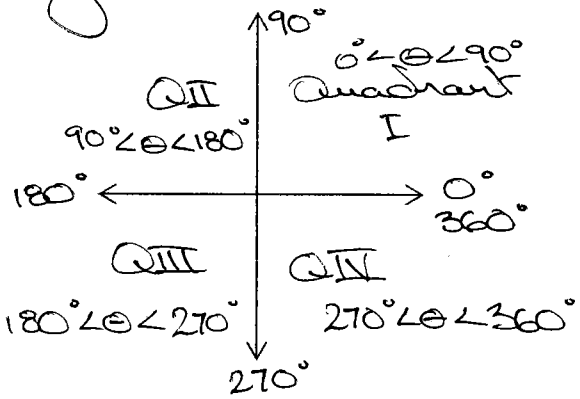
Angles in Standard Position, $0^o \leq \theta \leq 360^o$

On a Cartesian plane (x, y), you can generate an angle by rotating a ray about the origin. The starting position of the ray, along the positive x-axis, is the initial arm of the angle. The final position, after a rotation about the origin, is the terminal arm of the angle.

An angle is said to be an angle in standard position if its vertex is at the origin of a coordinate grid and its initial arm coincides with the positive x-axis.

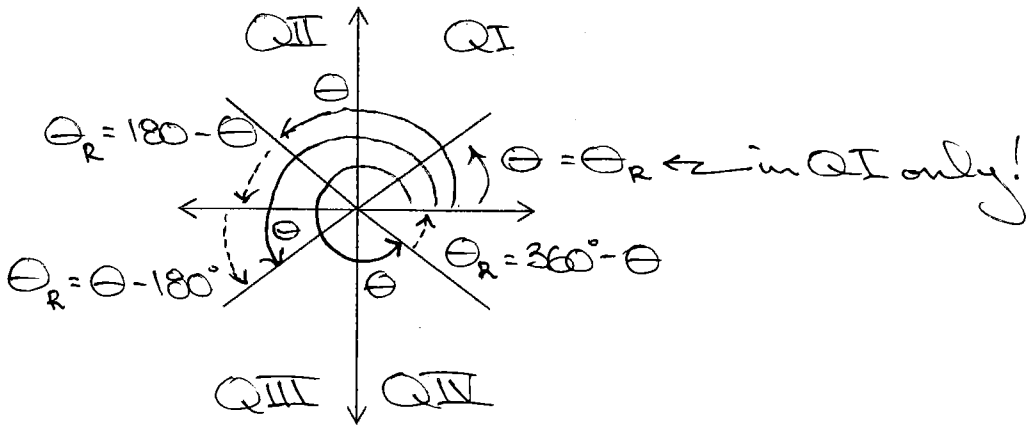
note: positive angles are always measured counter clock-wise.





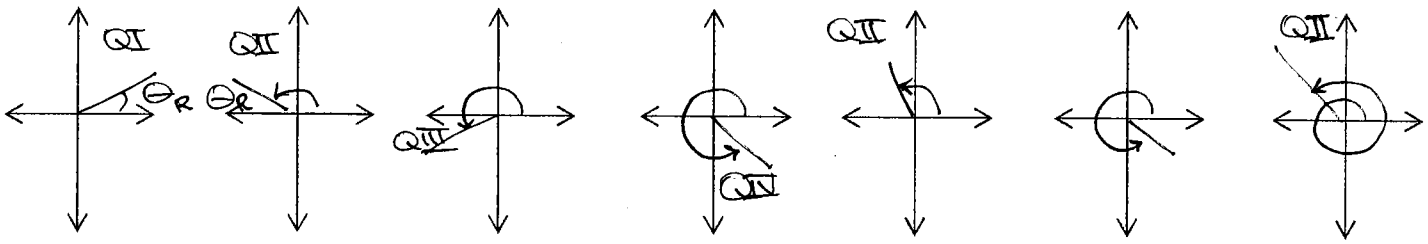
Reference Angle θ_R : the acute angle ($\angle$ 90°) whose vertex is at the origin and whose arms are: the terminal arm of the angle and the x-axis.

In General,



Example 1: Find the reference angle of the following principal angles:

20° 160° 200° 340° 108° 337° 500°

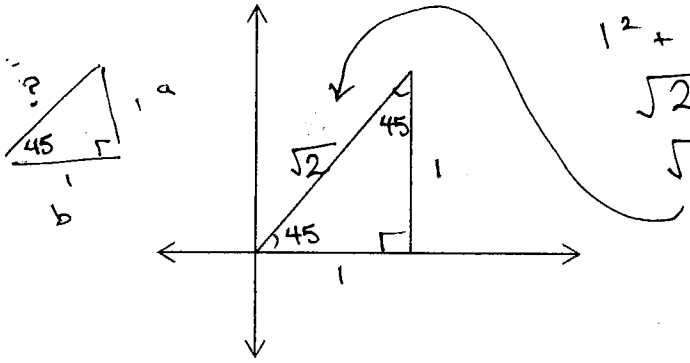


$\theta_R = 20°$ $\theta_R = 180-160 = 20°$ $\theta_R = 200-180 = 20°$ $\theta_R = 360-340 = 20°$ $\theta_R = 180-108 = 72°$ $\theta_R = 360-337 = 23°$ $\theta_R = 500-360 = 140$, $180-140$, $\theta_R = 40°$

Special Angles/Special Right Triangles

For angles of $30°, 60°, and\ 45°$, you can determine the exact value of the trigonometric ratios: Sin, Cos and Tan.

Type 1: $45°:45°:90°$ triangle.



$a^2+b^2=c^2$

$1^2+1^2=H^2$

$\sqrt{2}=\sqrt{H^2}$

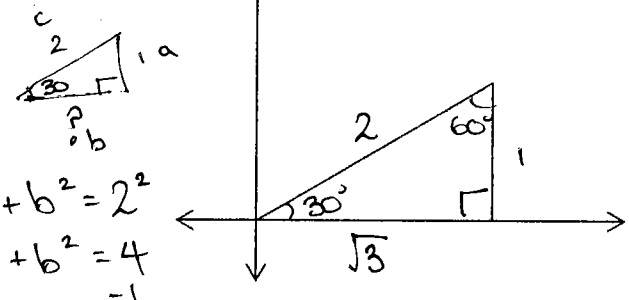
$\sqrt{2}=H$

$\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$ equivalent

$\tan 45° = \frac{1}{1} = 1$

Type 2: $30°:60°:90°$ triangle.



$1^2+b^2=2^2$

$1+b^2=4$

$-1 \quad -1$

$b^2=\sqrt{3}$

$b=\sqrt{3}$

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

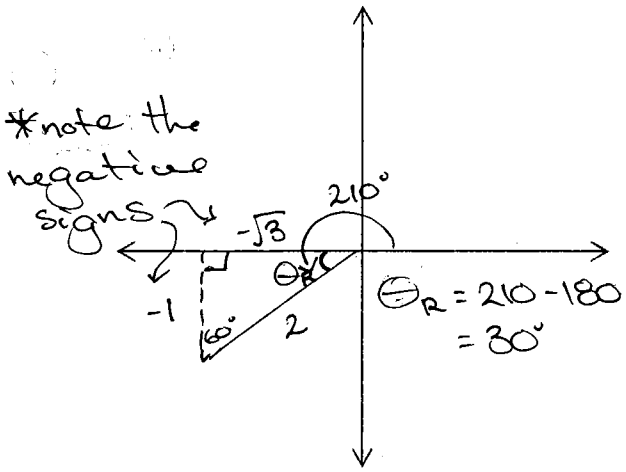
$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

Example 2: Determine the **exact ratios** for $Cos\ 210^o$ *and* $Tan\ 210^o$

↳ use reference angle & special triangle → 30°-60°-90°

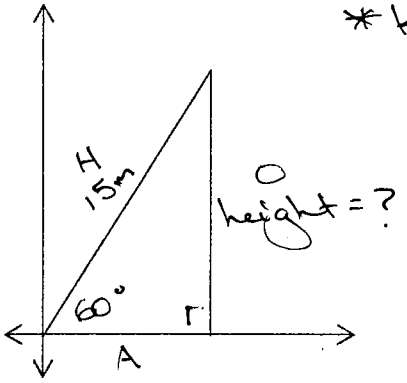


$\theta_R = 210 - 180$
$= 30°$

$\cos 210° = \frac{-\sqrt{3}}{2}$

$\tan 210° = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

Example 3: A crane lifts its arm to an angle of 60 ° from the ground (horizontal). If its arm is 15m long what is the **exact value** of its vertical displacement?



* know hypotenuse & want to know opposite.

↳ $\sin 60° = \frac{O}{H} = \frac{height}{15m}$

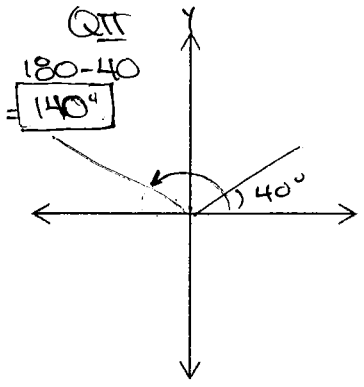
* know $\sin 60° = \frac{\sqrt{3}}{2}$ from special Δ.

↳ set up ratio: $\frac{height}{15} = \frac{\sqrt{3}}{2}$

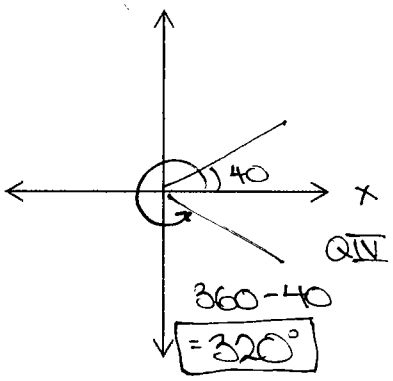
$height = \frac{15\sqrt{3}}{2}$ meters.

Example 4: Determine the angle in standard position when an angle of 40° is reflected

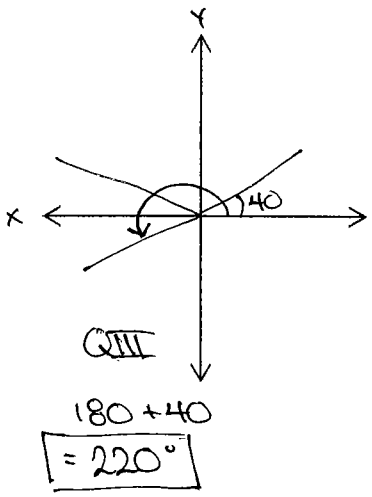
a) In the y-axis



b) In the x-axis



c) In the y-axis and then in the x-axis



Homework: Page 83 #2, 3abc, 4 cd, 5, 6abc, 7ab, 8, 9, 13, 15