

2.2 Factoring Special Polynomials

A. Factoring PERFECT SQUARE TRINOMIALS

A perfect square trinomial has the form: ax^2+bx+c

Perfect square trinomials have the following characteristics:

- ax^2 and c must be perfect squares (1, 4, 9, x^2 , x^4 , etc....)
- bx is equal to twice the product of the square roots of ax^2 and c terms.

Example 1: Verify that the following trinomials are perfect square trinomials.

a) $4x^2 + 12x + 9$

$$\begin{array}{ccc} \sqrt{4x^2} & & \sqrt{9} \\ \swarrow & & \downarrow \\ 2(2x) & (3) & = 12x \end{array}$$

b) $9x^2 - 6x + 1$

$$\begin{array}{ccc} \sqrt{9x^2} & & \sqrt{1} \\ \swarrow & & \downarrow \\ 2(3x) & (1) & = 6x \end{array}$$

Example 2: Factor the following trinomials

a) $4x^2 + 4x + 1$

check: $\sqrt{4x^2}$ $\sqrt{1}$

$$\begin{array}{ccc} 2(2x) & (1) & = 4x \\ \swarrow & & \searrow \\ (2x & + & 1)^2 \end{array}$$

b) $4x^2 - 12x + 9$

check: $\sqrt{4x^2}$ $\sqrt{9}$

$$\begin{array}{ccc} 2(2x) & (3) & = 12x \\ \swarrow & & \searrow \\ (2x & - & 3)^2 \end{array}$$

c) $4x^2 - 20x + 25x^2$

check: $\sqrt{4}$ $\sqrt{25x^2}$

$$\begin{array}{ccc} 2(2) & (5x) & = 20x \\ \swarrow & & \searrow \\ (2 & - & 5x)^2 \end{array}$$

d) $x^2 - 2x + 1$

check: $\sqrt{x^2}$ $\sqrt{1}$

$$\begin{array}{ccc} 2(x) & (1) & = 2x \\ \swarrow & & \searrow \\ (x & - & 1)^2 \end{array}$$

e) $100x^2 - 140xy + 49y^2$

check: $\sqrt{100x^2}$ $\sqrt{49y^2}$

$$\begin{array}{ccc} 2(10x) & (7y) & = 140xy \\ \swarrow & & \searrow \\ (10x & - & 7y)^2 \end{array}$$

B. Factoring the DIFFERENCE OF SQUARES

The difference of squares has the form: $ax^2 - c$

The difference of squares has the following characteristics:

- There are only two terms in the polynomial (binomial) *no bx (middle) term*
- Each term is a perfect square
- Second term must be subtracted from the first term

Example 3: Factor the following binomials

a) $16x^2 - 25$

check: $\sqrt{16x^2}$ $\sqrt{25}$

$(4x)$ (5)

$= (4x - 5)(4x + 5)$

b) $49n^2 - 121$

$= (7n - 11)(7n + 11)$

c) $x^4 - 1$

$= (x^2 + 1)(x^2 - 1)$

$= (x^2 + 1)(x - 1)(x + 1)$

d) $5x^4 - 80y^4$

$= 5(x^4 - 16y^4)$

$= 5(x^2 + 4y^2)(x^2 - 4y^2)$

$= 5(x^2 + 4y^2)(x - 2y)(x + 2y)$

e) $(x - 3)^2 - 16$

$= (x - 3 + 4)(x - 3 - 4)$

$= (x + 1)(x - 7)$

f) $w^2 - 5$

$= (w - \sqrt{5})(w + \sqrt{5})$