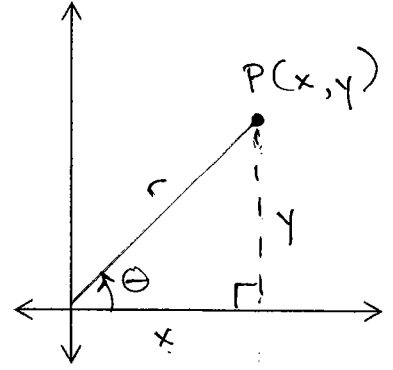


2.2 Trigonometric Ratios of Any Angle θ (part 1)

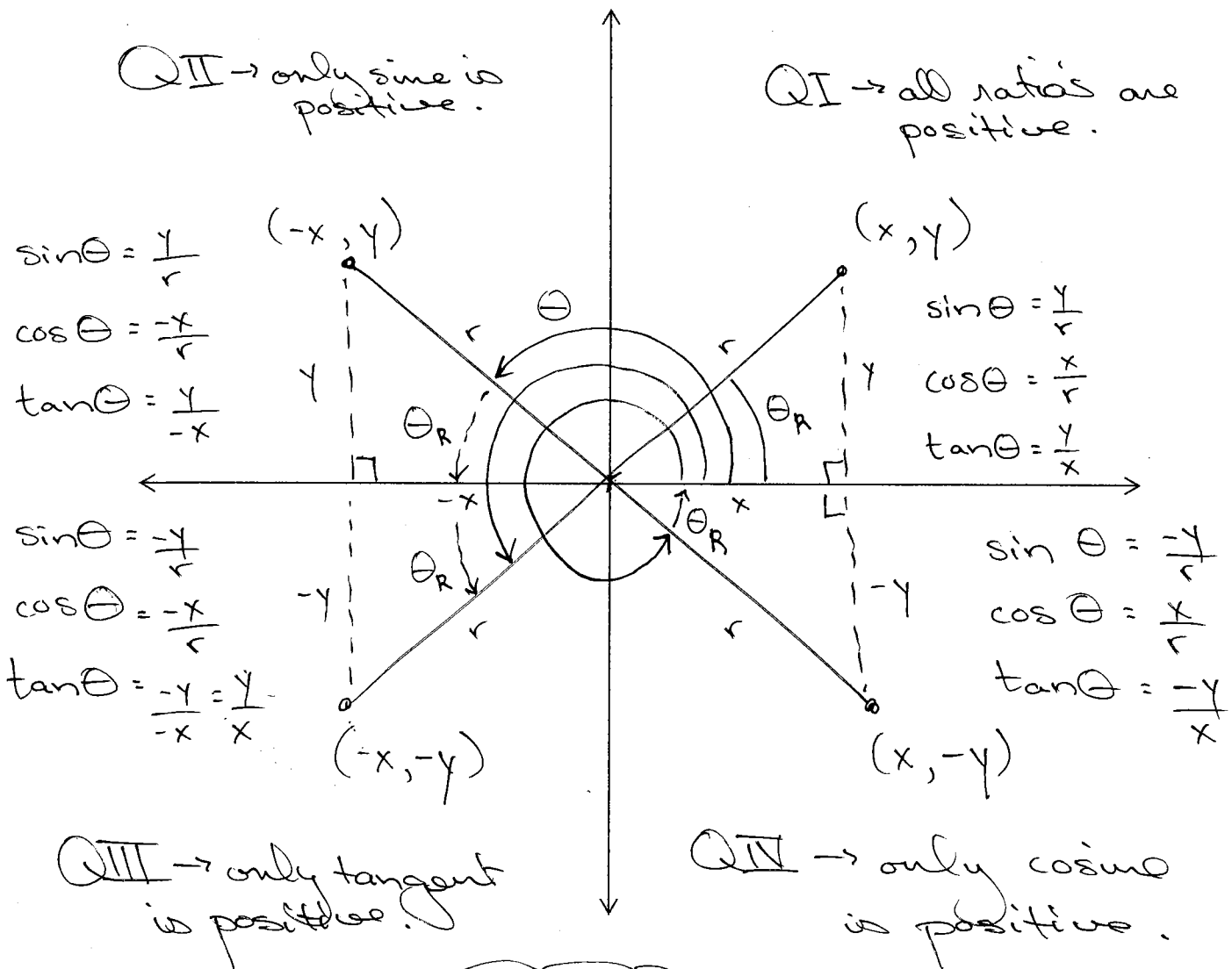
Suppose θ is any angle in standard position, and point $P(x, y)$ is any point on its terminal arm, at a distance r from the origin.



You can use a reference triangle to determine the three primary trig ratios in terms of x , y and r .

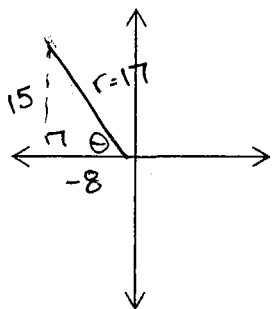
$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

Complete the chart below to identify the signs (positive or negative) of the ratios.



A-S-T-C rule

Example 1: Point $P(-8, 15)$ lies on the terminal arm of an angle θ , in standard position. Determine the exact trig ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$.



$$a^2 + b^2 = c^2$$

$$(-8)^2 + (15)^2 = r^2$$

$$\sqrt{289} = \sqrt{r^2}$$

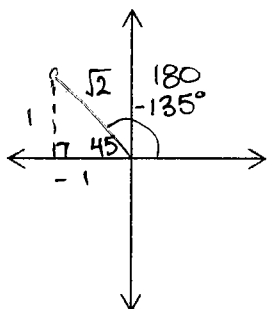
$$17 = r$$

$$\sin\theta = \frac{15}{17}$$

$$\cos\theta = \frac{-8}{17}$$

$$\tan\theta = \frac{15}{-8}$$

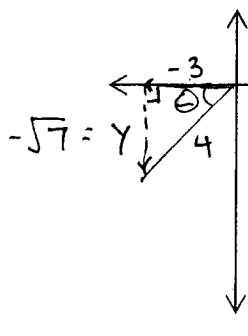
Example 2: Determine the exact value of $\cos 135^\circ$. (use special right triangle)



$$\cos 135^\circ = -\frac{1}{\sqrt{2}}$$

Example 3: Suppose θ is an angle in standard position with terminal arm in quadrant III, and

$\cos\theta = -\frac{3}{4}$. What are the exact values of $\sin\theta$, and $\tan\theta$?



$$a^2 + b^2 = c^2$$

$$y^2 + (-3)^2 = 4^2$$

$$y^2 + 9 = 16$$

$$y^2 = 7$$

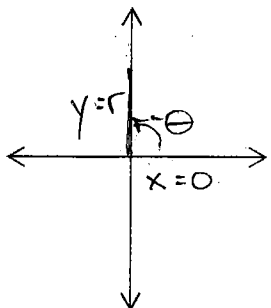
$\sqrt{y^2} = \sqrt{7}$ ← y is negative in QIII!

$$\sin\theta = -\frac{\sqrt{7}}{4}$$

$$\tan\theta = \frac{-\sqrt{7}}{-3} = \frac{\sqrt{7}}{3}$$

Example 4: Determine the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ when the terminal arm of quadrant angle θ coincides with the positive y -axis, $\theta = 90^\circ$.

(**quadrant angle:** an angle in standard position whose terminal arm lies on one of the axes)



$$\sin 90^\circ = \frac{y}{r}$$

$$= \frac{y}{y}$$

$$= 1$$

$$\cos 90^\circ = \frac{x}{r}$$

$$= \frac{0}{r}$$

$$= 0$$

$$\tan 90^\circ = \frac{y}{x}$$

$$= \frac{y}{0}$$

$$= \text{undefined}$$