2.3 Factoring Composite Quadratic Trinomials and Higher Ordered Polynomials

A. Composite Quadratic Trinomials

A composite quadratic trinomial has the form: $a(x-d)^2+b(x-d)+c$

A composite quadratic trinomial has the following characteristics:

- There are three terms in the polynomial
- The first and second terms have the same binomial
- The first term's binomial is squared

Steps for factoring a composite quadratic trinomial

1) Replace the binomial with a different variable $\longrightarrow (x-d)=y$
2) Factor the resulting trinomial
   - If a = 1, factor the simple trinomial
   - If a does not = 1, factor by decomposition / rainbow split

Example 1: Factor

let $y=(x-3)$

a) $(x-3)^2+6(x-3)+8$

$4+2=6$
$4\cdot 2=8$

y^2+6y+8
$=(y+4)(y+2)$ ← replace binomial & simplify
$=((x-3)+4)((x-3)+2)$
$=\boxed{(x+1)(x-1)}$

let $y=(x+1)$

b) $(x+1)^2+(x+1)-72$

$-8+9=1$
$-8\cdot 9=-72$

y^2+y-72
$=(y-8)(y+9)$
$=((x+1)-8)((x+1)+9)$
$=\boxed{(x-7)(x+10)}$

let $y=(x-1)$

c) $2(x-1)^2-5(x-1)-3$

$-6+1=-5$
$-6\cdot 1=-6$

$=2y^2-5y-3$ $\quad -6$
$=(2y^2-6y)(+y-3)$
$=2y(y-3)+1(y-3)$
$=(y-3)(2y+1)$
$=((x-1)-3)(2(x-1)+1)$
$=(x-4)(2x-2+1)$
$=(x-4)(2x-1)$

let $y=(x+2)$

d) $12(x+2)^2+5(x+2)-2$

$8+-3=5$
$8\cdot -3=-24$

$=12y^2+5y-2$ $\quad -24$
$=(12y^2+8y)(-3y-2)$
$=4y(3y+2)-1(3y+2)$
$=(3y+2)(4y-1)$
$=(3(x+2)+2)(4(x+2)-1)$
$=(3x+6+2)(4x+8-1)$
$=(3x+8)(4x+7)$

B. Higher Ordered Polynomials

A higher ordered polynomial, in Pre-Calculus 11, will be limited to the following:

- The highest order will be even → ex. $x^4 = (xx)(xx) = (x^2)(x^2)$
- Mostly trinomials

Example: Factor

a) $x^4 + 4x^2 + 3$

$\underline{3} + \underline{1} = 4$

$\underline{3} \cdot \underline{1} = 3$

$(x^2+3)(x^2+1)$

check by expanding:

$(x^2+3)(x^2+1)$

$= x^4 + x^2 + 3x^2 + 3$

$= x^4 + 4x^2 + 3$

b) $x^6 - x^3 - 12$

$\underline{-4} + \underline{3} = -1$

$\underline{-4} \cdot \underline{3} = -12$

$(x^3-4)(x^3+3)$

check by expanding:

$(x^3-4)(x^3+3)$

$= x^6 + 3x^3 - 4x^3 - 12$

$= x^6 - x^3 - 12$

c) $6x^4 - 11x^2 - 35$

-210

$\underline{-21} + \underline{10} = -11$

$\underline{-21} \cdot \underline{10} = -210$

$= (6x^4 - 21x^2)(+10x^2 - 35)$

$= 3x^2(2x^2-7) + 5(2x^2-7)$

$= (2x^2-7)(3x^2+5)$

d) $x^4 - 81$

difference of squares

$= (x^2+9)(x^2-9)$ ← again!

$= (x^2+9)(x+3)(x-3)$

HW: 2.3 WS