

Foundations and Precalculus 10

2.6 – Factoring Special Polynomials

Ex#1: Perfect Square Trinomial

$$(a + b)^2 = (a+b)(a+b)$$

$$= a^2 + ab + ab + b^2$$

$$= a^2 + 2ab + b^2$$

perfect squares → a^2 and b^2

double the product of the square root of first & last term.

$$\frac{6}{6} + \frac{6}{6} = 12$$

$$\frac{6}{6} \cdot \frac{6}{6} = 36$$

$$4x^2 + 12x + 9 =$$

$$= (4x^2 + 6x + 6x + 9)$$

$$= 2x(2x+3) + 3(2x+3)$$

$$= (2x+3)(2x+3)$$

Same as

$$= (2x+3)^2$$

Pattern Observed:

- a) The first and third terms are perfect squares: $\sqrt{4x^2} = 2x$ and $\sqrt{9} = 3$
- b) The second term is **twice** the product of the squares of terms a and c : $2(2x \cdot 3) = 2(6x) = 12x$
 a) check first/last terms

You try:

$$\sqrt{36x^2 + 12x + 1}$$

b) Check middle term → $2(6x \cdot 1) = 2(6x) = 12x$ ✓

$$36x^2 + 12x + 1 = (6x + 1)^2$$

$$\sqrt{16 + 56x + 49x^2}$$

$$2(4 \cdot 7x) = 2(28x) = 56x$$

$$= (4 + 7x)^2$$

* make your own Perfect Square trinomial.

Ex#2: Factor Trinomials with two variables : no different!

$$5x^2 - 13xy + 6y^2$$

$$= (5x^2 - 10xy + 6y^2 - 3xy)$$

$$= 5x(x-2y) - 3y(x-2y)$$

$$= (x-2y)(5x-3y)$$

$$\frac{-10}{-10} + \frac{-3}{-3} = -13$$

$$\frac{-10}{-10} \cdot \frac{-3}{-3} = 30$$

Factors of 30 / Sum Factors

check: $(x-2y)(5x-3y)$

$$= 5x^2 - 3xy - 10xy + 6y^2$$

$$= 5x^2 - 13xy + 6y^2$$
 ✓

shorter method: check for...
 ① both perfect squares
 ② separated by minus symbol.

Ex#3: Difference of Squares

$$\sqrt{x^2 - 16}$$

$$= (x+4)(x-4)$$

always one plus (one minus).

→ binomial: no middle term
 → both terms perfect squares. separated by minus.

longer method

$$x^2 + 0x - 16$$

$$= (x^2 + 4x)(4x - 16)$$

$$= x(x+4) - 4(x+4)$$

$$= (x+4)(x-4)$$

$$\frac{4}{4} + \frac{-4}{-4} = 0$$

$$\frac{4}{4} \cdot \frac{-4}{-4} = -16$$

check: $(x+4)(x-4)$

$$= x^2 - \cancel{4x} + \cancel{4x} - 16$$

$$= x^2 - 16 \checkmark$$

#4) $\sqrt{49 - c^2}$

$$= (7+c)(7-c)$$

What if...
 #5) $81x^2 + 49$ ⇒ $x^2 + 16 \rightarrow x^2 + 0x + 16$

~~$+ - = 0$~~
 ~~$- \cdot - = 16$~~ * nothing multiplies to 16 and adds to 0.

∴ $x^2 + 16$ is **not factorable**.

#6) $3x^2 - 300$ GCF = 3

$$= 3(\sqrt{x^2 - 100})$$

$$= 3(x+10)(x-10)$$

check: $3(x+10)(x-10)$

$$= 3(x^2 - \cancel{10x} + \cancel{10x} - 100)$$

$$= 3(x^2 - 100)$$

$$= 3x^2 - 300 \checkmark$$

#7) $\sqrt{81x^4 - 1}$

$$= (9x^2+1)(9x^2-1)$$

another difference of squares

$$= (9x^2+1)(3x+1)(3x-1)$$

* always check if you can factor further.