3.1 Factors and Multiples of Whole Numbers

A **prime number** is a number that is only divisible by one and itself.

e.g. 2, 3, 5, 7, 11, ...

A **factor** of a number is any number that will divide evenly into it.

e.g. 12: 1, 2, 3, 4, 6, 12

The **prime factorization** of a number is that number written as a product of its prime factors.

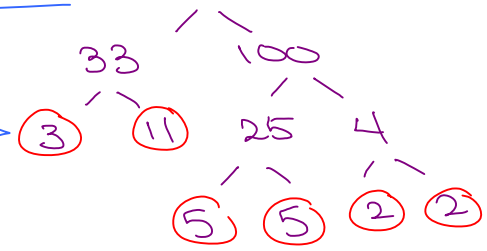
e.g. $12 = 2 \cdot 2 \cdot 3$ or $12 = (2)(2)(3)$

Ex. #1: Write the prime factorization of 3300.

Steps:
1) factor tree
2) write multiplication statement.

Method 1

circle when prime →



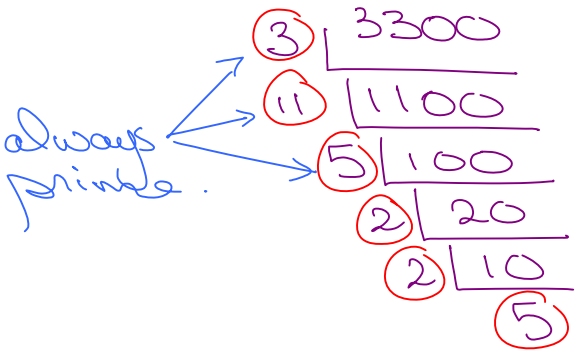
$3300 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$

or

$3300 = 2^2 \cdot 3 \cdot 5^2 \cdot 11$

Method 2

always prime.



use this bracket to list a set of numbers.

The prime factors of 3300 are $\{2, 3, 5, 11\}$

The prime factorization of 3300 is: $(2)(2)(3)(5)(5)(11)$

* if a number doesn't end in an even number, then it is not divisible by any multiple of 2.

Record any divisibility rules that you can recall here:

* ending in an even number is divisible by 2.
* ending in a zero or five is divisible by 5.
* sum of the digits is divisible by 3 then whole number is divisible by 3.

The **greatest common factor** (GCF) is the largest number that will divide evenly into a group of numbers.

Ex. #2: Determine the greatest common factor of 138 and 198.

Steps:
1) factor tree for each number
2) multiply common factors.

Method 1 - write out all factors of each.

138: 1, 2, 3, 6, 23, 46, 69, 138

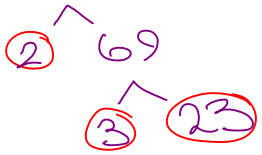
198: 1, 2, 3, 6, 11, 18, 33, 66, 99, 198

* takes to long!

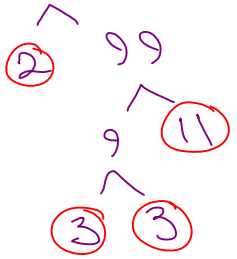
Better!

* *Method 2*

$138 = 2 \cdot 3 \cdot 23$



$198 = 2 \cdot 3^2 \cdot 11$



common factors: one 2 & one 3

$GCF = 2 \cdot 3$

$= 6$

The greatest common factor is: 6

The **least common multiple** (LCM) is the smallest number that a group of numbers will divide into.

Ex. #3: Determine the least common multiple of 18, 20, and 30.

Method 1 - List multiples of each number.

18: 18, 36, 54, 72, 90, 108...

20: 20, 40, 60, 80, 100, 120...

30: 30, 60, 90, 120, 150...

* takes too long!

Better* *Method 2*

$18 = 2 \cdot 3^2$ (2, 9; 9 → 3, 3)

$20 = 2^2 \cdot 5$ (4, 5; 4 → 2, 2)

$30 = 2 \cdot 3 \cdot 5$ (3, 10; 10 → 2, 5)

Steps

1) factor tree for number.

2) multiply all factors, but common factors only use highest exponent.

$LCM = 2^2 \cdot 3^2 \cdot 5$

$= 180$

The least common multiple of 18, 20, and 30 is: 180