

### 3.3 Completing the Square - Part 1

From grade 10, we can expand a squared binomial to find a pattern for factoring perfect square trinomials.

$$\begin{aligned} & (x+6)^2 \\ &= (x+6)(x+6) \\ &= x^2 + 12x + 36 \end{aligned}$$

perfect square trinomial

Recall: vertex form  $\rightarrow y = a(x-p)^2 + q$   
 standard form  $\rightarrow y = ax^2 + bx + c$

A technic called "Completing the Square" is used to change equations from standard form into vertex form. Vertex form is preferred because it is much easy to graph than standard form.

**Example 1:** Rewrite the following quadratic functions in vertex form.

a)  $y = (x^2 + 12x)$

bring neg. term out of brackets by multiplying what's in front of brackets

$$y = 1(x^2 + 12x + 36 - 36)$$

$$y = (x^2 + 12x + 36) - 36$$

$$y = (x + 6)^2 - 36$$

vertex  $(-6, -36)$

b)  $y = (x^2 - 4x) - 3$

~~$y = x^2 - 4x$~~

$$y = (x^2 - 4x + 4 - 4) - 3$$

$$y = (x^2 - 4x + 4) - 4 - 3$$

$$y = (x - 2)^2 - 7$$

vertex  $(2, -7)$

① group terms including variable (1st two)

② add a constant term that makes it perfect square trinomial; do not change original function subtract the constant too.

$$\left\{ c = \left(\frac{b}{2}\right)^2 \right\} = \left(\frac{12}{2}\right)^2$$

③ factor perfect square trinomial & cut

collect like terms

Example 2: Rewrite the following quadratic functions in vertex form.

a)  $y = 2x^2 + 8x$

~~$y = -5x^2 + 4x - 1$~~

must find GCF  
of terms with  
variable.

$$y = 2(x^2 + 4x)$$

$$y = 2(x^2 + 4x + 4 - 4)$$
$$= 2(x^2 + 4x + 4) - 8$$
$$= 2(x + 2)^2 - 8$$

c)  $y = 5x^2 + 30x + 41$

$$y = 5(x^2 + 6x) + 41$$

$$y = 5(x^2 + 6x + 9 - 9) + 41$$

$$y = 5(x^2 + 6x + 9) - 45 + 41$$

$$y = 5(x + 3)^2 - 4$$