3.3 Completing the Square – Part 2

vertex

Example 1: Determine the maximum or minimum value of the function and the value of ***x*** at which it occurs: → convert standard into vertex form

complete the square!

$\left(\frac{b}{2}\right)^2 = \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

a) $y = x^2 - 6x - 4$

$y = (x^2 - 6x + 9 - 9) - 4$

$y = (x^2 - 6x + 9) - 9 - 4$

$y = (x - 3)^2 - 13$

b) $y = -2x^2 + \frac{5}{2}x$

GCF = -2

$y = -2\left(x^2 - \frac{5}{4}x\right)$

$y = -2\left(x^2 - \frac{5}{4}x + \frac{25}{64} - \frac{25}{64}\right)$

$y = -2\left(x^2 - \frac{5}{4}x + \frac{25}{64}\right) - \frac{25}{32}$

$y = -2\left(x - \frac{5}{8}\right)^2 + \frac{25}{32}$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-5}{4 \cdot 2}\right)^2 = \left(\frac{-5}{8}\right)^2 = \frac{25}{64}$

GCF $= -\cancel{2}\left(-\frac{25}{\cancel{64}_{32}}\right) = +\frac{25}{32}$

Example 2: Verify in two different ways that the two algebraic forms represent the same function:

$y = -4(x+1)^2 + 6$ and $y = -4x^2 - 8x + 2$

same

same

GCF = -4

↳ change to standard

$y = -4(x+1)(x+1) + 6$

$y = -4(x^2 + x + x + 1) + 6$

$y = -4(x^2 + 2x + 1) + 6$

$y = -4x^2 - 8x - 4 + 6$

$y = -4x^2 - 8x + 2$

↳ change to vertex form

$y = -4(x^2 + 2x) + 2$

$y = -4(x^2 + 2x + 1 - 1) + 2$

$y = -4(x^2 + 2x + 1) + 4 + 2$

$y = -4(x + 1)^2 + 6$

$\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = (1)^2 = 1$

Example 3: A theatre company has 300 season ticket subscribers. The directors have decided to increase the price of a season ticket from the current price of $400. A survey of the subscribers has determined that for every $20 increase in price, 10 subscribers would not renew their season tickets.

a) What price would maximize the revenue from season tickets?

Revenue = (price)(# of tickets)

Let n be the # of price increases.

$$\text{price} = 400 + 20n$$

$$\#\text{ of tickets} = 300 - 10n$$

$$R = (400 + 20n)(300 - 10n)$$

$$R = 120000 \underbrace{- 4000n + 6000n}_{} - 200n^2$$

$$R = 120000 + 2000n - 200n^2$$

rearrange

~~**Example 4:** A sporting goods store sells flip-flops for $8. At this price their weekly sales are approximately 100 pairs. Research says that for every $2 increase in price, the manager can expect the store to sell five fewer pairs of flip-flops.~~

~~a) What quadratic function would represent this situation?~~

~~b) Determine the maximum revenue (at what price & how many pairs of flip-flops)?~~

$$R = (\underbrace{-200}_{\text{GCF}}n^2 + 2000n) + 120000$$

$$R = -200(n^2 - 10n + 25 - 25) + 120000$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$$

$$R = -200(n^2 - 10n + 25) + 5000 + 120000$$

$$R = -200(n - 5)^2 + 125000$$

vertex $(\underbrace{5}_{\#\text{ of price increases}}, 125000)$ max. revenue = $\boxed{\$125000}$

Practice: p. 192 # 6bd, 7bc, 8c, 15a, 18ab, 19ab

$$\text{price} = 400 + 20(5) = \boxed{\$500}$$