1) Simplify

a) $\frac{6x}{x-5} - \frac{240}{x^2-2x-15}$

$\frac{x+3}{x+3}\frac{6x}{x-5} - \frac{240}{(x-5)(x+3)}$

$\frac{6x^2+18x-240}{(x-5)(x+3)}$

$\frac{6(x^2+3x-40)}{(x-5)(x+3)}$

$\frac{6(x+8)(x-5)}{(x-5)(x+3)} = \frac{6(x+8)}{x+3}$

Solve

b) $\frac{1}{x} + \frac{1}{x-3} = \frac{x-2}{x-3}$

LCD = $x(x-3)$

NPV $x \neq 0, 3$

$x(x-3)\left(\frac{1}{x} + \frac{1}{x-3}\right) = \left(\frac{x-2}{x-3}\right)x(x-3)$

$x-3+x = x(x-2)$

$2x-3 = x^2-2x$

$-2x+3 \quad -2x+3$

$0 = x^2-4x+3$

$0 = (x-1)(x-3)$

$x=1$ ✓ $x=3$ ✗ extraneous

2) Solve graphically

a) $-x^2+4x+5 \leq 0$

$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$-x^2+4x+5 \leq 0$

$-1 \quad -1$

$x^2-4x-5 \geq 0$

$(x^2-4x+4-4)-5 \geq 0$

$(x-2)^2-9 \geq 0$

$f(x)$ above (pos) x-axis

or

$-(x-2)^2+9 \leq 0$

same

same

$\{x \mid x \leq -1, x \geq 5, x \in \mathbb{R}\}$

Solve algebraically

b) $-x^2+4x+5 \leq 0$

$-1 \quad -1$

$x^2-4x-5 \geq 0$

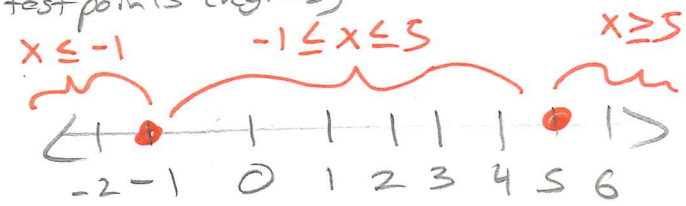
$(x+1)(x-5) \geq 0$

$x=-1, 5$

roots

- factor (roots)
- test points (regions)

$x \leq -1$ | $-1 \leq x \leq 5$ | $x \geq 5$

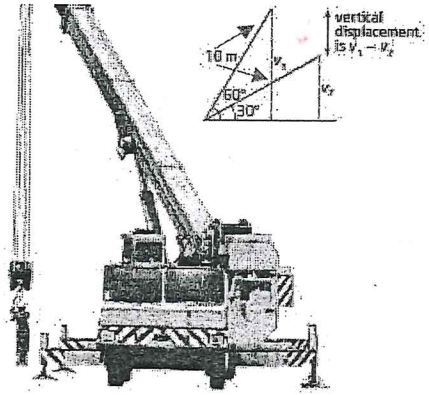


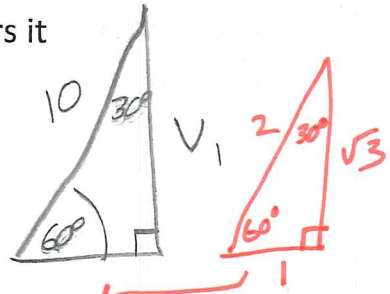
$x=-2$	$x=0$	$x=6$
$x^2-4x-5 \geq 0$	$(0)^2-4(0)-5 \geq 0$	$(6)^2-4(6)-5 \geq 0$
$(-2)^2-4(-2)-5 \geq 0$	$0-0-5 \geq 0$	$36-24-5 \geq 0$
$4+8-5 \geq 0$	$-5 \geq 0$	$7 \geq 0$
$12-5 \geq 0$	x	✓
$7 \geq 0$		

$\therefore \{x \mid x \leq -1, x \geq 5, x \in \mathbb{R}\}$

2) A 10m crane arm lifts material on to a roof.

Determine the EXACT VALUE ratio of the vertical displacement of the end of the boom when the operator lowers it from 60° *to* 30°.

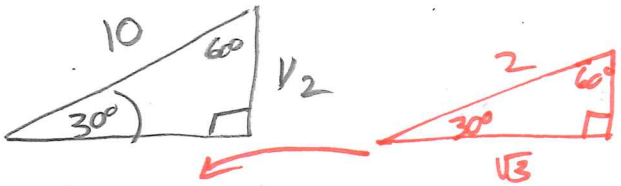




$\sin 60° = \frac{V_1}{10}$

$10\left(\frac{\sqrt{3}}{2}\right) = \left(\frac{V_1}{10}\right)10$

$V_1 = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$



$\sin 30° = \frac{V_2}{10}$

$10\left(\frac{1}{2}\right) = \left(\frac{V_2}{10}\right)10$

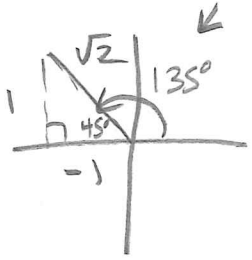
$V_2 = 5$

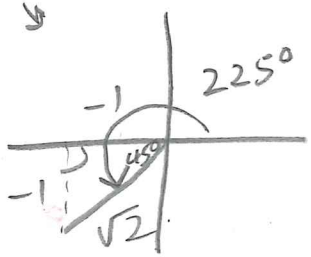
$$\frac{V_1 - V_2}{5\sqrt{3} - 5}$$

or $5(\sqrt{3} - 1)$

3) Prove the following without the use of a calculator (hint: it involves special angle triangles)

a) $\cos 135° = \sin 225°$





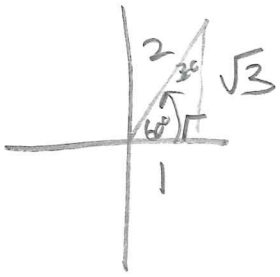
$\cos 135° = \frac{-1}{\sqrt{2}}$ *

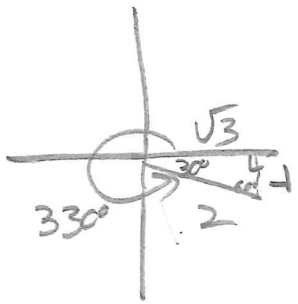
$\sin 225° = \frac{-1}{\sqrt{2}}$ *

Same ratio

$\therefore \cos 135° = \sin 225°$

b) $\sin 60° = \cos 330°$





$\sin 60° = \frac{\sqrt{3}}{2}$ *

$\cos 330° = \frac{\sqrt{3}}{2}$ *

same ratio

$\therefore \sin 60° = \cos 330°$