

5.5 notes

Tuesday, March 10, 2020 1:17 PM

PreCalc 11

5.5-Solving Radicals using Algebra

Review: $\sqrt{2} * \sqrt{2} = 2$
 $(\sqrt{2})^2$

$\sqrt{x} * \sqrt{x} = x$
 $(\sqrt{x})^2$

$\sqrt{x-3} * \sqrt{x-3} = x-3$
 $(\sqrt{x-3})^2$

Like solving by graphing, solving through algebra means to find the x-value(s) where the two functions cross or intersect.

To solve algebraically we must

- ① set radicand to ≥ 0
- ② solve for variable.

- 1) Identify the restriction(s) on the variable under the root. This is the same as determining the domain in which the function exists.
- 2) Isolate the radical on one side of the equation.
- 3) Square both sides of the equation to remove the radical.
- 4) Solve for x.
- 5) Make sure the x-value is within the domain of the function. Check your solution(s) back into any equation with a radical to make sure there are no extraneous solns.

① satisfies the restriction

* must check every time, every solution!

Solve using algebra:

Rest. $2x - 1 \geq 0$
 $+1 \quad +1$
 $\frac{2x}{2} \geq \frac{1}{2}$
 $x \geq \frac{1}{2}$

$5 + \sqrt{2x-1} = 12$
 $-5 \quad -5$
 $(\sqrt{2x-1})^2 = (7)^2$
 $2x - 1 = 49$
 $+1 \quad +1$
 $\frac{2x}{2} = \frac{50}{2}$
 $x = 25$

Check: $x = 25$

$5 + \sqrt{2x-1} = 12$
 $5 + \sqrt{2(25)-1} = 12$
 $5 + \sqrt{49} = 12$
 $5 + 7 = 12$
 $12 = 12 \checkmark$

Rest. $(\cancel{5}) \frac{3x}{\cancel{5}} \geq 0(5)$

monomial

$x \geq 0$

Solve using algebra:

$$\begin{array}{r} -8 + \sqrt{\frac{3x}{5}} = -2 \\ +8 \quad +8 \end{array}$$

$$\left(\sqrt{\frac{3x}{5}}\right)^2 = (6)^2$$

$$(5) \frac{3x}{\cancel{5}} = 36(5)$$

$$\frac{3x}{3} = \frac{180}{3}$$

Check: $x = 60$ $x \geq 0$

$$-8 + \sqrt{\frac{3(60)}{5}} = -2$$

$$-8 + \sqrt{36} = -2$$

$$-8 + 6 = -2$$

$$-2 = -2 \checkmark$$

$$\begin{array}{r} 3x \geq 0 \\ \cancel{3} \quad \cancel{3} \\ x \geq 0 \end{array}$$

$$\begin{array}{r} x + \sqrt{x-1} = 3 \\ -x \quad -x \end{array}$$

Rest. $x - 1 \geq 0$

$x \geq 1$

$$(\sqrt{x-1})^2 = (3-x)^2$$

$$x-1 = (3-x)(3-x)$$

$$x-1 = 9 - 3x - 3x + x^2$$

$$\begin{array}{r} \cancel{x} - \cancel{1} = 9 - 6x + x^2 \\ -x + 1 \quad +1 \quad -x \end{array}$$

$$0 = 10 - 7x + x^2 \leftarrow \text{rearrange}$$

$$0 = x^2 - 7x + 10$$

Solve this quadratic:

- factor

- quadratic formula

$$\begin{array}{l} - + \quad - = -7 \\ - \cdot \quad - = 10 \end{array}$$

$$0 = (x-5)(x-2)$$

$$\begin{array}{r} x-5=0 \\ +5 \quad +5 \end{array}$$

$$\begin{array}{r} x-2=0 \\ +2 \quad +2 \end{array}$$

Check: $x = 5$

$x = 2$ $x \geq 1$

$$x + \sqrt{x-1} = 3$$

$$x + \sqrt{x-1} = 3$$

$$(5) + \sqrt{5-1} = 3$$

$$(2) + \sqrt{2-1} = 3$$

$$5 + \sqrt{4} = 3$$

$$2 + \sqrt{1} = 3$$

$$5 + 2 = 3$$

$$3 = 3 \checkmark$$

7 \neq 3 extraneous solution

① Rest. $2\cancel{4} + 7 \geq 0$

① Rest. $2x + 7 \geq 0$

$$\begin{array}{r} 2x = -4 + 2\sqrt{2x+7} \\ +4 \quad +4 \end{array}$$

$$\frac{2x+4}{2} = \frac{2\sqrt{2x+7}}{2}$$

$$(x+2)^2 = (\sqrt{2x+7})^2$$

$$(x+2)(x+2) = 2x+7$$

$$x^2 + 4x + 4 = 2x + 7$$

$$x^2 + 2x - 3 = 0$$

a = -1 b = 2 c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

Calculator

* 2 radicals:

$$7 + \sqrt{3x} = \sqrt{5x+4} + 5$$

isolate the worse one

$$(2 + \sqrt{3x})^2 = (\sqrt{5x+4})^2$$

$$(2 + \sqrt{3x})(2 + \sqrt{3x}) = 5x + 4$$

isolate $4 + 2\sqrt{3x} + 2\sqrt{3x} + 3x = 5x + 4$

$$\begin{array}{r} 4 + 4\sqrt{3x} + 3x = 5x + 4 \\ -4 \quad -3x \quad -3x \quad -4 \end{array}$$

$$(4\sqrt{3x})^2 = (2x)^2$$

$$16(3x) = 4x^2$$

$$48x = 4x^2$$

$$\begin{array}{r} -48x \quad -48x \end{array}$$

$$0 = 4x^2 - 48x$$

$$0 = x^2 - 12x$$

$$0 = x(x - 12)$$

Check $x = 0$

Check $x = 12$

$$7 + \sqrt{3x} = \sqrt{5x+4} + 5$$

$$7 + \sqrt{3(0)} = \sqrt{5(0)+4} + 5$$

$$7 + \sqrt{0} = \sqrt{4} + 5$$

$$7 = 7$$

$$7 + \sqrt{3x} = \sqrt{5x+4} + 5$$

$$7 + \sqrt{3(12)} = \sqrt{5(12)+4} + 5$$

$$7 + \sqrt{36} = \sqrt{64} + 5$$

$$7 + 6 = 8 + 5$$

$$\frac{2x}{2} \geq \frac{-7}{2}$$

Check:

$x = 1$

$$2(1) = -4 + 2\sqrt{2(1)+7}$$

$$2 = -4 + 2\sqrt{9}$$

$$2 = -4 + 6$$

$$2 = 2 \checkmark$$

Check:

~~$x = -3$~~

$$2(-3) = -4 + 2\sqrt{2(-3)+7}$$

$$-6 = -4 + 2\sqrt{1}$$

$$-6 = -4 + 2$$

$$-6 = -2 \times$$

Rest. * with 2 radicals!

$$3x \geq 0$$

$x \geq 0$

$$5x + 4 \geq 0$$

$$\frac{5x}{5} \geq \frac{-4}{5}$$

$$x \geq -\frac{4}{5}$$

analyze:

$$x = 0$$

$$\sqrt{3(0)} = \sqrt{0} \checkmark$$

$$\sqrt{5(0)+4} = \sqrt{4} \checkmark$$

$$x = \frac{4}{5}$$

$$\sqrt{3(\frac{4}{5})} = \sqrt{\frac{12}{5}}$$

Not OK!

Other 2 radical example that doesn't require doing steps #2-5 twice:

$$\sqrt{3x} + \sqrt{5x+4} = 0$$

$$\begin{array}{r} -\sqrt{3x} \quad -\sqrt{3x} \end{array}$$

$$(\sqrt{5x+4})^2 = (\sqrt{3x})^2$$

$$5x+4 = 3x$$

$$\begin{array}{r} -3x \quad -3x \end{array}$$

$$2x+4 = 0$$

$$\begin{array}{r} -4 \quad -4 \end{array}$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$x = -2$$

Check: $x = -2$

$$7 + \sqrt{0} = \sqrt{4} + 5$$

$$7 = 2 + 5$$

$$7 = 7 \checkmark$$

$$7 + \sqrt{36} = \sqrt{64} + 5$$

$$7 + 6 = 8 + 5$$

$$13 = 13 \checkmark$$