

## 6.1 Rational Expressions

A rational expression is an algebraic fraction with a polynomial in the numerator and/or denominator.

ex.  $\frac{1}{2x}$  ;  $\frac{x+2}{x^2-3x+1}$  ;  $\frac{x^2+1}{2}$

**Non-permissible values** are any value(s) of the variable that make the denominator equal to zero. (npv) *set each factor  $\neq 0$  and solve for x*

**Example 1:** Determine the non-permissible values of the following rational expressions:

a)  $\frac{2x}{x-2}$

$$\begin{array}{r} x-2 \neq 0 \\ +2 \quad +2 \\ \hline x \neq 2 \end{array}$$

b)  $\frac{5}{2xy^2}$

$$\begin{array}{l} 2xy^2 \neq 0 \\ x \neq 0 \quad y^2 \neq 0 \\ y \neq 0 \end{array}$$

c)  $\frac{5x}{x^2-3x+2}$

$$\begin{array}{l} x^2-3x+2 \neq 0 \text{ -factor} \\ (x-1)(x-2) \neq 0 \\ \begin{array}{r} x-1 \neq 0 \quad x-2 \neq 0 \\ +1 \quad +1 \quad \quad +2 \quad +2 \\ \hline x \neq 1 \quad \quad x \neq 2 \end{array} \end{array}$$

where possible  $\rightarrow$  ① factor numerator & denominator  
 ② list npv  
 ③ cancel common factors

**Example 2:** Simplify the following rational expressions:

a)  $\frac{x+2}{x^2+4x+4}$  - factor  $-+ = 4$   
 $-x = -4$

$$\frac{x+2}{(x+2)(x+2)}$$

①

$$\begin{array}{l} x+2 \neq 0 \quad x+2 \neq 0 \\ -2 \quad -2 \\ \hline x \neq -2 \end{array}$$

same

②

$$\frac{\cancel{x+2}}{\cancel{(x+2)}(x+2)}$$

③

$$= \boxed{\frac{1}{x+2}}$$

b)  $\frac{m^3t^1}{m^2t^4}$  - divide powers with like terms, subtract exponents.

① can't factor so go to ②

npv.  $m^2t^4 \neq 0$

$$\begin{array}{l} \sqrt{m^2} \neq 0 \quad \sqrt[4]{t^4} \neq 0 \\ m \neq 0 \quad t \neq 0 \end{array}$$

$$\frac{m^3t^1}{m^2t^4} = m^1t^{-3} = \boxed{\frac{m}{t^3}}$$

$$c) \frac{3x-6}{2x^2+x-10}$$

$$= \frac{3(x-2)}{(2x+5)(x-2)}$$

A.P.V.  $x \neq 2, -\frac{5}{2}$

$$= \boxed{\frac{3}{2x+5}}$$

$+1 \rightarrow 5(-4)$   
 $-20 \rightarrow$   
 $(2x^2-4x)(+5x-10)$   
 $2x(x-2)+5(x-2)$

reorder  $\rightarrow$  want  
 term first  
 $\frac{6-2m}{m^2-9} = \frac{-2(m+3)}{(m+3)(m-3)}$

n.p.v.  $m \neq \pm 3$

$$= \boxed{-\frac{2}{m-3}}$$

**Example 3:** Rational expressions with **pairs** of non-permissible values

a) Simplify the following rational expression:  $\frac{16x^2-9y^2}{8x-6y}$

$$= \frac{(4x+3y)(4x-3y)}{2(4x-3y)}$$

$$= \boxed{\frac{4x+3y}{2}}$$

n.p.v.  $x \neq \frac{3y}{4}$

b) Evaluate the expression for  $x = 2.6$  and  $y = 1.2$

$$\frac{4(2.6) + 3(1.2)}{2}$$

$$= \boxed{7}$$