6.4 Solving Rational Equations

- You can **solve** a rational equation by **multiplying both sides** of the equation by a **common denominator** (clear denominators). This eliminates the fractions from the equation. Then solve the resulting equation.

- **Check** that the potential roots (solutions, zeros, ...) satisfy the original equation, are NOT non-permissible values and are realistic in the context of a word problem.

Identify the LCD: $12\left(\frac{?}{3}+\frac{?}{4}-\frac{?}{2}\right)$ $x^3\left(\frac{?}{x^2}+\frac{?}{x}+\frac{?}{x^3}\right)$ $(x+2)(x-2)\left(\frac{?}{x+2}+\frac{?}{x-2}-\frac{?}{x^2-4}\right)$ $(x+2)(x-2)$

Steps: ① Factor / ② Non-permissible Values / ③ Identify LCD / ④ Eliminate Denominators / ⑤ Solve / Check

Example: Solve the following rational equations.

a) $\frac{3}{x^2}+\frac{4}{x}=\frac{-1}{1}$ x^2

npv: $x \neq 0$

LCD $= x^2$

$\frac{3x^2}{x^2}+\frac{4x^2}{x}=\frac{-1x^2}{1}$

$3+4x=-1x^2$

$+1x^2 \quad +1x^2$

$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x=-3$ $\quad$ $x=-1$

check $x=-3$

$\frac{3}{(-3)^2}+\frac{4}{(-3)}=\frac{-1}{1}$

$\frac{3}{9}-\frac{4}{3}=-1$

$-\frac{3}{3}=-1$ ✓

$x=-1$

$\frac{3}{(-1)^2}+\frac{4}{(-1)}=\frac{-1}{1}$

$\frac{3}{1}-\frac{4}{1}=-1$

$-1=-1$ ✓

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b) $\frac{4}{x}-\frac{3}{x+1}=1$ $x(x+1)$

npv: $x \neq 0, -1$

LCD $= x(x+1)$

$\frac{4(x)(x+1)}{x}-\frac{3(x)(x+1)}{x+1}=x(x+1)$

$[4(x+1)]-[3(x)]=x(x+1)$

$(4x+4)-(3x)=x^2+x$

$x+4=x^2+x$

$-x \quad -4 \quad -x \quad -4$

$0=x^2-4$

$0=(x+2)(x-2)$

$x=\pm 2$

check $x=2$

$\frac{4}{2}-\frac{3}{2+1}=1$

$2-1=1$ ✓

$x=-2$

$\frac{4}{(-2)}-\frac{3}{-2+1}=1$

$-2-(-3)=1$

$1=1$ ✓

c) $\frac{4x-1}{x+2}-\frac{x+1}{x-2}=\frac{x^2-4x+24}{x^2-4}$

$(x+2)(x-2)$

$(x+2)(x-2)$

LCD $=(x+2)(x-2)$

npv: $x \neq \pm 2$

$(4x-1)(x-2)-(x+1)(x+2)=x^2-4x+24$

$(4x^2-9x+2)-(x^2+3x+2)=x^2-4x+24$

$4x^2-9x+2-x^2-3x-2-x^2+4x-24=0$

$2x^2-8x-24=0$

$2(x^2-4x-12)=0$

$2(x-6)(x+2)=0$

$x=6 \quad x=-2$ npv

check $x=6$

$\frac{4(6)-1}{6+2}-\frac{6+1}{6-2}=\frac{6^2-4(6)+24}{6^2-4}$

$\frac{23}{8}-\frac{14}{8}=\frac{36}{32}$ 9 8

$\frac{9}{8}=\frac{9}{8}$ ✓

d) $\frac{9}{x-3}-\frac{4}{x-6}=\frac{18}{x^2-9x+18}$

$(x-6)(x-3)$

$(x-6)(x-3)$

LCD: $(x-6)(x-3)$

npv $x \neq 3, 6$

$[9(x-6)]-[4(x-3)]=18$

$(9x-54)-(4x-12)=18$

$9x-54-4x+12=18$

$5x-42=18$

$+42 \quad +42$

$\frac{5x}{5}=\frac{60}{5}$

$x=12$

check $x=12$

$\frac{9}{12-3}-\frac{4}{12-6}=\frac{18}{12^2-9(12)+18}$

$\frac{x^3}{x3}-\frac{4^2}{6\,3}=\frac{1}{3}$

$\frac{1}{3}=\frac{1}{3}$ ✓

Practice: p 348 # 1-4