7.2 Quadratic Inequalities (2 var)

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7.2 Quadratic Inequalities in Two Variables

A quadratic inequality in two variables may be in one of the following forms:

$y > ax^2 + bx + c$ $\quad y \geq ax^2 + bx + c$ where a, b & c are real numbers.

$y < ax^2 + bx + c$ $\quad y \leq ax^2 + bx + c$

An inequality in two variables describes a region in the Cartesian plane. Any point (x, y) that satisfies the inequality is a solution to the inequality. The set of all points that satisfy the inequality is called the solution set or solution region.

The parabola related to the quadratic equality $y = ax^2 + bx + c$ is the boundary that divides the Cartesean plane into two possible solution regions.

- When the inequality sign is $\leq$ or $\geq$ the points on the boundary are included and the graph is a solid curve.
- When the inequality sign is $<$ or $>$ the points on the boundary are not included and the graph is a dashed curve.

Steps to graphing a quadratic inequality in two variables:

1) Determine if boundary is solid ($\leq$; $\geq$) or dashed ($<$; $>$).
2) Graph boundary parabola.
3) Choose test point: not a point on the boundary.
4) Shade appropriate region: where TP satisfies the inequality (TRUE)

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Example 1: Graph $y < -2(x-3)^2+1$

dashed

down

vertex $(3,1)$ $a=-2$

stretched by 2

direction

x	y	$-2y$
± 1	1	$-2(1)=-2$
± 2	4	$-2(4)=-8$

TP $(0,0)$

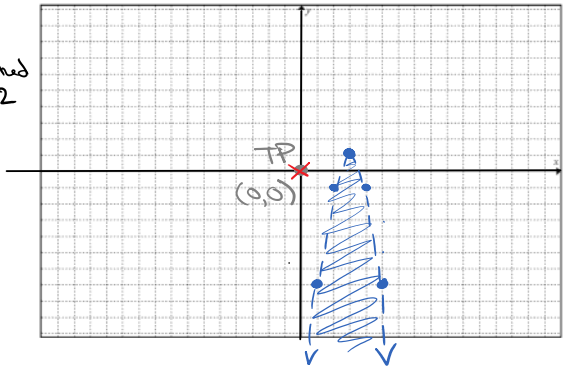
$(0) < -2((0)-3)^2+1$

$0 < -2(-3)^2+1$

$0 < -2(9)+1$

$0 < -18+1$

$0 < -17$ false



Example 2: Graph $y \geq x^2-4x-5$

solid

$\left(\frac{-4}{2}\right)^2=(-2)^2=4$

TP $(1,0)$

$(0) \geq (1)^2-4(1)-5$

$0 \geq 1-4-5$

$0 \geq -8$

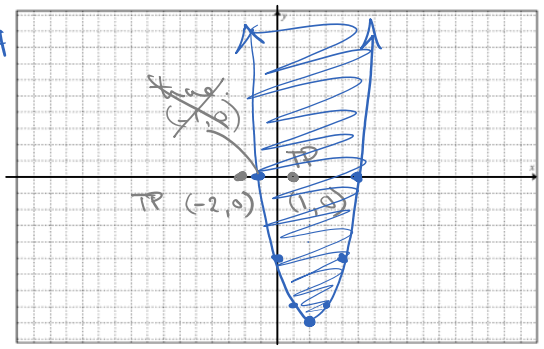
true!

$(x^2-4x)-5$

$1(x^2-4x+4-4)-5$ (+4 and −4)

$y \geq (x-2)^2-9$

vertex $(2,-9)$ $a=1$



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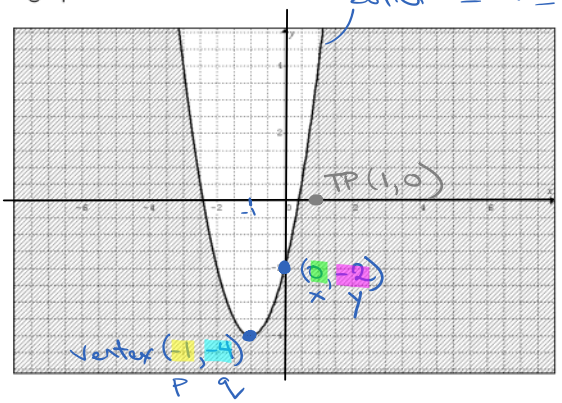
Example 3: Write an inequality to represent the graph.



$y \square a(x-p)^2+q$

$(-2) \square a((0)-(-1))^2+(-4)$

$-2 \square a(1)^2-4$

$-2 \square a-4$

$+4 \quad +4$

$2 \square a$

$y \square 2(x-(-1))^2+(-4)$

$y \square 2(x+1)^2-4$

$(0) \square 2((1)+1)^2-4$

$\square 2(2)^2-4$

$\square 8-4$

$\square \leq 4$

$y \leq 2(x+1)^2-4$ or

$y \leq 2(x+1)(x+1)-4$

$y \leq 2(x^2+x+x+1)-4$

$y \leq 2(x^2+2x+1)-4$

$y \leq 2x^2+4x+2-4$

$y \leq 2x^2+4x-2$

Example 4: Write an inequality to represent the graph.

Sub vertex & one other point on parabola into vertex form: $y=a(x-p)^2+q$

$(0) = a((0)-(2))^2+(4)$

Simplify & solve for 'a'

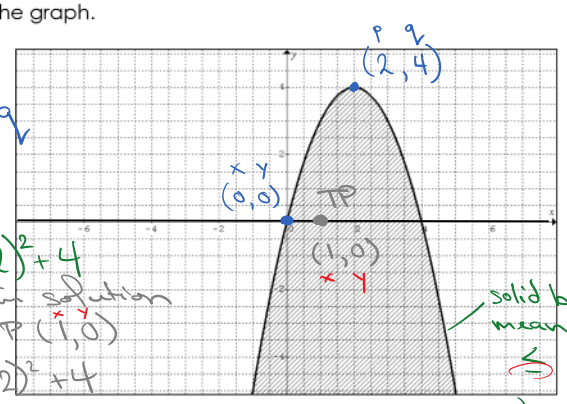
$0 = a(-2)^2+4$

$0 = 4a+4$

$-4 \quad -4$

$\frac{-4}{4} = \frac{4a}{4}$

$-1 = a$



$y \leq -(x-2)^2+4$

choose TP in solution region: TP (1,0)

$(0) \square -((1)-2)^2+4$

$\square -(-1)^2+4$

$\square -1+4$

$\square < 3$

solid boundary line means either: ≤ or ≥

$y \leq -(x-2)^2+4$

Practice: p. 496 # 3ab, 4ab, 6ab, 7ab

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