8.2 Compound interest

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8.2 – Compound Interest

Two kinds of Canada Savings Bonds (CSB) are regular and compound bonds. Regular Canada Savings Bonds earn simple interest that is deposited into the owner's bank account each year. Compound Canada Savings Bonds earn compound interest and the total amount of the bond is paid when it is cashed.

Consider the growth of a $500 CSB of each type at and interest rate of 5% over a 5-year period: Use simple interest.

Regular CSB			
Year	P($)	I($)	A($)
1	500	30	530
2	500	50	550
3	500	75	575
4	500	100	600
5	500	125	625

Compound CSB			
Year	P($)	I($)	A($)
1	500	30	530
2	530	26.5	556.50
3	556.50	27.83	584.33
4	584.33	29.22	613.55
5	613.55	30.68	644.23

Which CSB type would you choose and why?

The compound CSB because it generates more interest and therefore is worth more.

When interest is earned or paid on interest, the interest compounds. This is known as compounding interest and the formula used to calculate it is: generates interest on interest

final amount

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

Where,

P = principal amount

r = annual nominal interest rate (as a decimal)

n = number of times the interest is compounded per year

t = number of years

$\boxed{I = A - P}$

Types of compounding	Number of compounding periods
Annually	n = 1
Semi-annually	2
Monthly	12
Bi-weekly	$\left(\frac{52}{2}\right)$ 26
Weekly	52
Daily	365
Quarterly	4

twice a year

every other week

Example 1: $7000 is invested in a 6 year GIC compounded quarterly at a rate of 5% per annum. Determine the value of the investment at the end of the term.

P = 7000
r = 0.05
n = 4
t = 6
A = ?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 7000\left(1 + \frac{.05}{4}\right)^{4 \cdot 6}$

$\boxed{A = \$9431.46}$

Example 2: RBC and TD offer the following investment opportunities for an initial investment of $P =$ \$10000:

$r_1 = 0.073$ $n_1 = 1$

1. • RBC pays interest at an annual rate of 7.3% compounded annually.
2. • TD pays interest at an annual rate of 7.2% compounded monthly.

$r_2 = 0.072$ $n_2 = 12$

a) Which bank provides the greater interest at the end of:

i. 1 year $t = 1$

$A_1 = 10000\left(1 + \frac{.073}{1}\right)^{1\cdot 1}$ $A_2 = 10000\left(1 + \frac{.072}{12}\right)^{12\cdot 1}$

$A_1 = \$10730$ $A_2 = \$10754.93$

\$24.93 more interest in ②

ii. 10 years $t = 10$

$A_1 = 10000\left(1 + \frac{.073}{1}\right)^{1\cdot 10}$

$A_1 = \$20230.06$

$A_2 = 10000\left(1 + \frac{.072}{12}\right)^{12\cdot 10}$

$A_2 = \$20500.18$

\$270.12 more in ②

b) What was the value of each investment at the end of 30 years. $t = 30$

$A_1 = 10000\left(1 + \frac{.073}{1}\right)^{1\cdot 30}$ $A_2 = 10000\left(1 + \frac{.072}{12}\right)^{12\cdot 30}$

$\boxed{A_1 = \$82792.63}$ $\boxed{A_2 = \$86153.53}$

Example 3: Roz received a loan for \$2500 for 4 years compounded bi-monthly and paid \$842.26 in interest. What was the annual rate of interest to the nearest tenth of a percent?

Solving for r so BEDMAS

$P = 2500$
$t = 4$
$n = 6$
$I = 842.26$
$r = ?$

$A = P + I$
$= 2500 + 842.26$
$A = 3342.26$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$\frac{3342.26}{2500} = \frac{\cancel{2500}\left(1 + \frac{r}{6}\right)^{6\cdot 4}}{\cancel{2500}}$

$\sqrt[24]{1.336\ldots} = \sqrt[24]{\left(1 + \frac{r}{6}\right)^{24}}$

$1.012\ldots - 1 = 1 + \frac{r}{6} - 1$

$6 \cdot 0.012\ldots = \frac{r}{6} \cdot 6$

$0.073 = r$

percent $\cdot 100$

$\boxed{7.3\% = r}$

Example 4: Andy wants to invest some money with the goal of having \$8000 in 5 years. The bank offers an annual rate of 5.7% compounded weekly. How much should Andy's initial investment be?

$P = ?$
$r = 0.057$
$n = 52$
$t = 5$
$A = 8000$

$\frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{P\cancel{\left(1 + \frac{r}{n}\right)^{nt}}}{\cancel{\left(1 + \frac{r}{n}\right)^{nt}}}$

$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$

$P = \frac{8000}{\left(1 + \frac{.057}{52}\right)^{52\cdot 5}}$

$\boxed{P = \$6017.05}$

Example 5: A GIC pays 3.49% interest, compounded annually. A principal of \$5000 is invested. Approximately how long (to the nearest whole number) will it take for the investment to:

Rule of 72:

$$\frac{72}{\text{Rate of Return}} = \text{Time for Investment to Double}$$

year

expressed as a percent

a) double?

$\frac{72}{3.49} = \boxed{21 \text{ years}}$

b) reach a value of \$20000?

It would need two doubling periods so $21 \cdot 2 = \boxed{42 \text{ years}}$

HW: 8.2 WS