8.3 – Annuities: Investments and Loans

Warm up:

Solve for x in the equation: $y = a(x-p)^2 + q$

$-q \qquad -q$

$\frac{y-q}{a} = \frac{\cancel{a}(x-p)^2}{\cancel{a}}$

$\pm\sqrt{\frac{y-q}{a}} = x - p$

$+p \qquad +p$

$p \pm \sqrt{\frac{y-q}{a}} = x$

An **annuity** is a series of equal deposits (or payments), equally distributed over time. The 2 common types of annuities that we'll look at are:

Investment annuity: where there is a certain principal deposited and then regular payments made over the course of the investment

Loan: where you make loan payments on a regular schedule (every month, year, quarter, etc.) and are paying interest on the loan

The amount of an annuity is also called the *future value* of the annuity.

FV or $A = \frac{R[(1+i)^n - 1]}{i}$

- A is the amount in dollars.
- R is the regular deposit or payment in dollars. → $R = \frac{Ai}{(1+i)^n - 1}$
- i is the interest rate per compounding period, as a decimal.
- n is the number of deposits or payments.

* these are simple annuities where how often payment is made = compounding period of interest rate.

Example 1: A regular deposit of $100 is invested in a savings account at the end of each month. The interest rate is 2% compounded monthly. What is the amount of the annuity at the end of 50 years?

$A = \frac{\left(100\left[\left(1+\frac{0.02}{12}\right)^{600} - 1\right]\right)}{\left(\frac{0.02}{12}\right)}$

$n = 12 \cdot 50$

$n = 600$

$R = 100$

$i = \frac{0.02}{12}$

$\boxed{A = \$102961.20}$

Example 2: Suppose you want to retire at age 60 with $1 000 000. At age 20, you start investing a monthly deposit in a stock portfolio that pays 4.6% interest compounded monthly. What should your monthly deposit be to achieve your goal?

40 yr

$A = 1000\,000$

$n = 12 \cdot 40 = 480$

$i = \frac{.046}{12}$

$R = ?$

$R = \frac{\left(1000000 \cdot \left(\frac{.046}{12}\right)\right)}{\left(\left(1 + \frac{.046}{12}\right)^{480} - 1\right)}$

$\boxed{R = \$726.78}$ per month.

An annuity can also be used to make *regular payments.* For example, a sum of money can be invested now to provide regular equal payments in the future that can be used to provide a source of income after you retire. The money that is invested now is called the present value of the annuity.

Present Value of an Annuity

The present value of an annuity is: $PV = \frac{R[1-(1+i)^{-n}]}{i}$

- PV is the present value in dollars.
- R is the regular payment in dollars.
- i is the interest rate per compounding period, as a decimal.
- n is the number of payments.

$$R = \frac{PVi}{1-(1+i)^{-n}}$$

The present value of a loan is the equivalent today of the future amount (or total amount) to be repaid.

Example 3: A person buys a sound system. She pays \$65 a month for 48 months at an interest rate of 12% compounded monthly. What is the present value of the loan?

$i = \frac{0.12}{12}$

$R = 65$

$n = 48$

$PV = ?$

$$PV = \frac{(65[1-(1+\frac{.12}{12})^{-48}])}{(\frac{.12}{12})}$$

$$\boxed{PV = \$2468.31}$$

Example 4: A person has a \$50 000 student loan. The loan is repaid over 7 years. The bank charges 6.5% interest compounded quarterly. What is the quarterly repayment on the loan?

$R = ?$

$i = \frac{.065}{4}$

$PV = 50000$

$n = 4 \cdot 7 = 28$

$$R = \frac{(50000(\frac{.065}{4}))}{(1-(1+(\frac{.065}{4}))^{-28})}$$

$\boxed{R = \$2236.89}$ per quarter

HW: 8.3 WS