## 8.4 The COSINE LAW

The Cosine Law describes the relationship between the <u>cosine</u> of an angle and the lengths of the three sides of <u>any</u> triangle.

For any $\Delta ABC$, where $a$, $b$, and $c$ are the <u>lengths</u> of the sides opposite to $\angle A, \angle B$ and $\angle C$ respectively, the cosine law states that:

find side

$a^2 = b^2 + c^2 - 2bc\cos A$ or $b^2 = a^2 + c^2 - 2ac\cos B$ or $c^2 = \cancel{a^2} + \cancel{b^2} - 2ab\cos C$

$\cos A = \dfrac{a^2 - b^2 - c^2}{-2bc}$  $\cos B = \dfrac{b^2 - a^2 - c^2}{-2ac}$  $-a^2 - b^2 \quad -\cancel{a^2} - \cancel{b^2}$

find angle

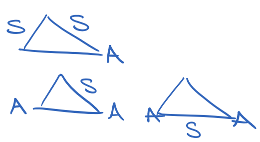
$c^2 - a^2 - b^2 = (\cancel{-2ab})\cos C$
$\overline{-2ab} \qquad \overline{-2ab}$

(Make sure you know how to solve equations properly!)  $\dfrac{c^2 - a^2 - b^2}{-2ab} = \cos C$

How do we know when to use **SOH CAH TOA** or **the SINE LAW** or **the COSINE LAW**?

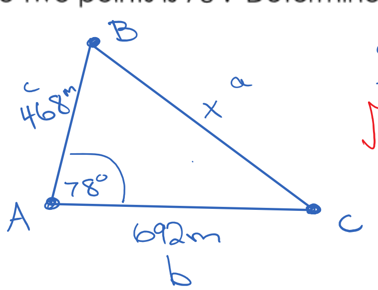
SOHCAHTOA
→ right Δ
→ 1 angle & 1 side

Sine Law
→ know/determine an angle & corresponding side
plus an angle or side.




Cosine Law
→ when Sine doesn't work


**Example 1:** A surveyor needs to find the length of a swampy area near Fish Lake. She sets her transit at point A. She measures the distance to one end of the swamp as 468 m (point B) and the distance to the other side/end of the swamp as 692 m (point C). The angle of sight betweer the two points is 78°. Determine the length of the swampy area, to the nearest tenth of a metre.



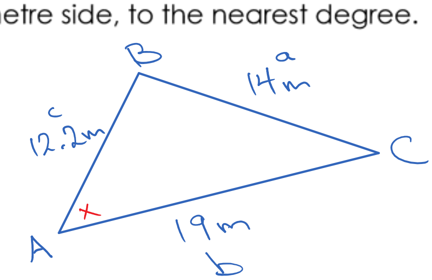

$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 692^2 + 468^2 - 2(692)(468)\cos 78°$

$\sqrt{x^2} \doteq \sqrt{563221.1}$

$x = 750.5$

**Example 2:** The Lions' Gate Bridge in Vancouver is strengthened by triangular braces. Suppose the braces lengths are 14 m, 19 m, and 12.2 m. Determine the measure of the angle opposite th 14 metre side, to the nearest degree.




$\cos A = \dfrac{a^2 - b^2 - c^2}{-2bc}$

$\cos x = \dfrac{(14^2 - 19^2 - 12.2^2)}{(-2(19)(12.2))}$

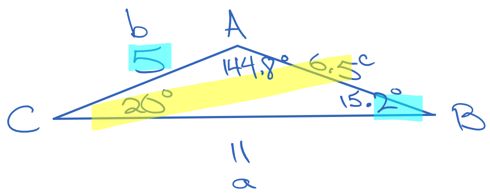
$\cos x = 0.6769\ldots$

$x = \cos^{-1}(0.6769\ldots)$

$\boxed{x = 47°}$

Pre-Calc 11

**Example 3**: In $\Delta ABC$ $a = 11,\ b = 5,\ \angle C = 20^\circ$. Determine the length of the unknown side and the measures of the 2 unknown angles, to the nearest tenth.




$c^2 = b^2 + a^2 - 2ba\cos C$

$c^2 = 5^2 + 11^2 - 2(5)(11)\cos 20^\circ$

$\sqrt{c^2} = \sqrt{42.6\ldots}$

$\boxed{c = 6.5}$

$\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{\sin 20^\circ}{6.5} = \frac{\sin B}{5}$

$\sin B \doteq 0.2619$

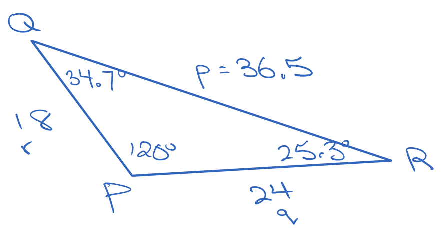
$B = \sin^{-1}(0.2619)$

$\boxed{\angle B = 15.2^\circ}$

$A = 180 - 20 - 15.2$

$\boxed{\angle A = 144.8^\circ}$

**Example 4**: In $\Delta PQR$ $q = 24,\ r = 18,\ \angle P = 120^\circ$. Solve the triangle.




$p^2 = q^2 + r^2 - 2qr\cos P$

$p^2 = 24^2 + 18^2 - 2(24)(18)\cos 120^\circ$

$\sqrt{p^2} = \sqrt{1332}$

$\boxed{p = 36.5}$

$\cos Q = \frac{24^2 - 18^2 - 36.5^2}{-2(18)(36.5)}$

$\cos Q = 0.8220\ldots$

$Q = \cos^{-1}(0.8220\ldots)$

$\boxed{\angle Q = 34.7^\circ}$

$R = 180 - 120 - 34.7$

$\boxed{\angle R = 25.7^\circ}$

**Practice**: P. 119 # 1 ab, 2 ab, 3 a, 4 a, 5a, 9, 10, 11, 17.