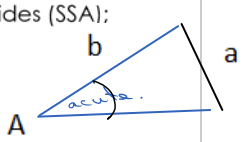
8.4 Sine Law (Part II) – The AMBUGUOUS CASE

In any triangle, if we have 2 sides and one angle opposite one of the sides (SSA);

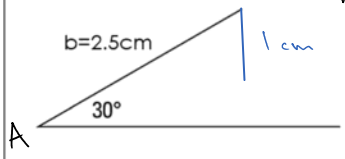
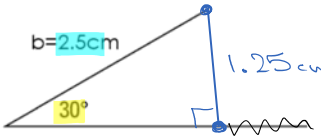
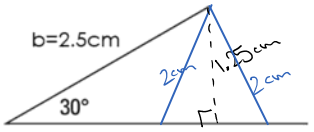


three possible outcomes may occur,

1) ***No*** triangle can be formed, so ***no solution*** is possible.
2) ***One*** triangle can be formed, so ***one solution*** is possible.
3) ***Two*** triangles can be formed (ambiguous case), so ***two solutions*** are possible.

In triangle ABC, $\angle A$ and side b are constant because they will be given. We need to look at the possible lengths of side a.

Using a ruler, complete the following triangles. In each triangle, you are given $\angle A = 30°$, and side $b = 2.5\ cm$. You are not given the length of side c or the measures of the other two angles.

Triangles	Need to Know
1. Let side $a = 1\ cm$ $h = b \cdot \sin A$ $h = 2.5 \cdot \sin 30°$ $= 1.25\ cm$ 	If $\angle A$ is acute and $a < b$, No triangle is formed.
2. Let side $a = 1.25\ cm$  1.25 cm = a = height of the triangle	If $\angle A$ is acute and $a = h$, 1 right $\triangle$ formed.
3. Let side $a = 2\ cm$ $h = 1.25\ cm$ 	If $\angle A$ is acute and $h < a < b$, 2 possible $\triangle$'s. * ambiguous case! * a is bigger than h and smaller than b

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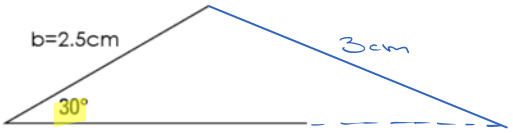
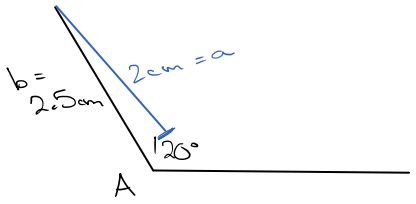
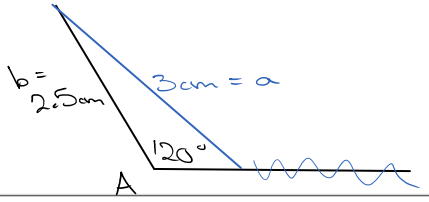
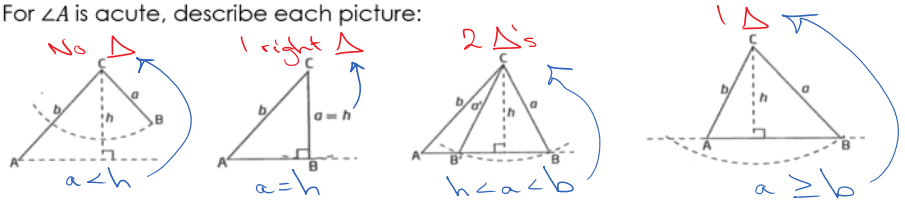
4. Let side $a = 3\ cm$  	If $\angle A$ is acute and $a \geq b$, 1 triangle formed.

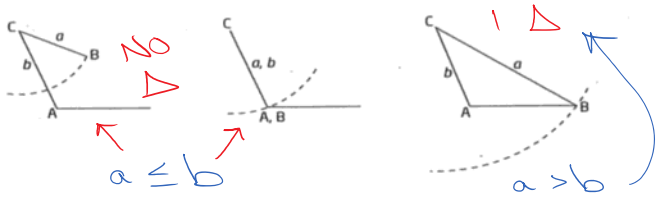
Using a ruler, complete the following triangles. In each triangle, you are given $\angle A = 120°$, and side $b = 2.5\ cm$. You are not given the length of side c or the measures of the other two angles.

Triangles	Need to Know
5. Let side $a = 2\ cm$  	If $\angle A$ is obtuse and $a \leq b$, No $\triangle$ formed
6. Let side $a = 3\ cm$  	If $\angle A$ is obtuse and $a > b$, 1 triangle formed.

For $\angle A$ is acute, describe each picture:



For $\angle A$ is obtuse, describe each picture:



h

$h = b \cdot \sin A$

Summary

$\angle A$ is acute		$\angle A$ is obtuse	
$a < b \cdot \sin A$	NO solution	$a < b$	NO solution
$a = b \cdot \sin A$	ONE solution (right triangle)	$a = b$	NO solution
$a \geq b$	ONE solution	$a > b$	ONE solution
$b \cdot \sin A < a < b$	TWO solutions		

Example 1: How many triangles are possible if: $\angle A = 26°$, $a = 15$ and $b = 19$ or ASS triangle.

① $h = b \cdot \sin A$
$= 19 \cdot \sin 26°$
$h = 8.3$

② $a \geq h$? Yes

③ $a < b$ and $a > h$
$15 < 19$

2 Δ's

Steps:
① $h = b \cdot \sin A$
② Is $a \geq h$?
③ Is $a < b$?

acute

obtuse ① $a > b$?

Example 2: How many triangles are possible if: $\angle A = 26°$, $a = 15$ and $b = 9$

① $h = b \cdot \sin A = 3.9$
$= 9 \cdot \sin 26°$

② $a > h$ ✓

③ $a > b$
$15 > 9$

1 Δ

Example 3: How many triangles are possible if: $\angle R = 26°, r = 15$ and $s = 19$

A → a → b

① $h = b \cdot \sin A$
$= 19 \cdot \sin 26°$
$h = 8.3$

② $a > h$ ✓

③ $a < b$
$15 < 19$

2 Δ's

Example 4: How many triangles are possible if: $\angle P = 126°, q = 15$ and $p = 15$

Example 5: How many triangles are possible if: $\angle P = 126°, q = 15$ and $p = 29$

Example 6: Solve triangle ABC: $\angle B = 126°$, $a = 25$ and $b = 17$, if possible.

A, b, a

$a > b$
$17 > 25$? NO.

No Δ formed

Practice: p. 108 # 6, 7, 8, 11, 12