

## 6.1 Rational Expressions

A rational expression is an algebraic fraction with a polynomial in the numerator and/or denominator.

**Non-permissible values** (or restrictions) are any value(s) of the variable that make the denominator equal to zero.

**Example 1:** Determine the non-permissible values of the following rational expressions:

a)  $\frac{2x}{x-2}$

b)  $\frac{5}{2xy^2}$

c)  $\frac{5x}{x^2-3x+2}$

**Example 2:** Simplify the following rational expressions:

a)  $\frac{x+2}{x^2+4x+4}$

b)  $\frac{m^3t}{m^2t^4}$

$$\text{c) } \frac{3x-6}{2x^2+x-10}$$

$$\text{d) } \frac{6-2m}{m^2-9}$$

**Example 3:** Rational expressions with **pairs** of non-permissible values

a) Simplify the following rational expression:  $\frac{16x^2-9y^2}{8x-6y}$

b) Evaluate the expression for  $x = 2.6$  and  $y = 1.2$

## 6.2 Multiplying and Dividing Rational Expressions

**Example 1:** Multiply the following rational expressions:

$$\text{a) } \frac{a^2 - a - 12}{a^2 - 9} \cdot \frac{a^2 - 4a + 3}{a^2 - 4a}$$

$$\text{b) } \frac{x^2 + x - 6}{x^2 + 2x - 15} \cdot \frac{x - 3}{x - 2}$$

**Example 2:** Divide the following rational expressions:

$$\text{a) } \frac{x^2 - x - 20}{x^2 - 6x} \div \frac{x^2 + 9x + 20}{x^2 - 12x + 36}$$

$$\text{b) } \frac{x^2 + 15x + 56}{x^2 - 3x - 54} \div \frac{x^2 + 6x - 16}{x^2 + 4x - 12}$$

### 6.3 Adding & Subtracting Rational Expressions Part 1

When adding or subtracting fractions or rational expressions, we need a common \_\_\_\_\_.

**Recall Fractions:**

$$\frac{2}{3} + \frac{5}{3}$$

$$\frac{3}{4} + \frac{2}{3}$$

**Example:** Adding or Subtracting Rational Expressions

a)  $\frac{2a}{c} - \frac{a-1}{c}$

b)  $\frac{2x}{x+4} + \frac{8}{x+4}$

c)  $\frac{x^2}{x-2} + \frac{3x}{x-2} - \frac{10}{x-2}$

d)  $\frac{2x}{xy} + \frac{4}{x^2}$

### 6.3 Adding & Subtracting Rational Expressions Part 2

**Example:** Adding or Subtracting Rational Expressions

$$\text{a) } \frac{y^2 - 20}{y^2 - 4} + \frac{y - 2}{y + 2}$$

$$\text{b) } \frac{x - 1}{x^2 + x - 6} - \frac{x - 2}{x^2 + 4x + 3}$$

## 6.4 Solving Rational Equations

- You can **solve** a rational equation by **multiplying both sides** of the equation by a **common denominator** (clear denominators). This eliminates the fractions from the equation. Then solve the resulting equation.
- **Check** that the potential roots (solutions, zeros, ...) satisfy the original equation, are NOT non-permissible values and are realistic in the context of a word problem.

Identify the LCD:  $\frac{?}{3} + \frac{?}{4} - \frac{?}{2}$        $\frac{?}{x^2} + \frac{?}{x} + \frac{?}{x^3}$        $\frac{?}{x+2} + \frac{?}{x-2} - \frac{?}{x^2-4}$

Steps: Factor / Non-permissible Values / Identify LCD / Eliminate Denominators / Solve / Check

**Example:** Solve the following rational equations.

a)  $\frac{3}{x^2} + \frac{4}{x} = \frac{-1}{1}$

b)  $\frac{4}{x} - \frac{3}{x+1} = 1$

$$c) \frac{4x-1}{x+2} - \frac{x+1}{x-2} = \frac{x^2-4x+24}{x^2-4}$$

$$d) \frac{9}{x-3} - \frac{4}{x-6} = \frac{18}{x^2-9x+18}$$