6.1 - Solving Linear Systems of Equations

A system of equations is two or more equations, considered together, involving the same variables. The ______________ is all the values of the variables that satisfy each. To solve means:

___________________________________________________________________________________________________________________________________
___________________________________________________________________________________________________________________________________.

1. **Linear-linear system** – A system of equations involving two linear equations involving the ____________ variables. A graph of this system involves two ______________.

The **solution** to a system of equations from a graph is the point(s) - or ordered pair(s) \((x, y)\) - where the two graphs ______________. Also called the ________________ points.

**How many solutions are possible?**

Linear-linear system

To solve a system of equations graphically:

1. ______________________________

2. ______________________________

3. ______________________________

**Example 1:** Solve the following system of equations graphically.

\[ 2x - 4y = 16 \]
\[ 4x + y = 5 \]
You try: Solve the following system of equations graphically.

\[ x - 2y + 2 = 0 \]
\[ x - y + 1 = 0 \]
Solving Linear Systems WS#1

Solve each system by graphing.

1) \[ y = \frac{1}{3}x - 3 \]
   \[ y = \frac{5}{3}x + 1 \]

2) \[ y = -4x - 2 \]
   \[ y = x + 3 \]

3) \[ y - 7x = 4 \]
   \[ -3y = -9 - 21x \]

4) \[ 2y - x = -2 \]
   \[ 16 - 8y = 2x \]
Solve each system by substitution.

5) \( y = -x + 1 \)
\[ x = \boxed{2} \]

6) \( y = -x + 4 \)
\[ y = 3x - 4 \]

7) \[-x + 1 = -\frac{1}{2}y \]
\[ 6 = 3y - 2x \]

8) \[-3x = 6 + 2y \]
\[ 0 = -4y + 8 - x \]
6.1 – Solving Linear Systems of Equations (Part 3)

Often it is useful to multiply one or both of the equations in a system to create a specific coefficient in front of one of the variables or to get rid of fractions.

Multiply to create -6 coefficient on the x variable:

\[ 2x + 3y = -5 \]

Multiply to get rid of all fractions:

\[ \frac{2}{3}x - \frac{3}{4}y = 1 \]

Steps to solve by ELIMINATION

1) Label each equation
2) Rearrange the equations to line up the variables
3) If necessary, multiply one or both of the equations by a constant (goal: when you +/- one of the variables cancel)
4) Add or subtract to eliminate one of the variables
5) Solve the equation
6) Substitute and solve for the remaining variable
7) Verify the solution(s)

Example 1: Solve the following system of equations

a) \[ \begin{align*}
y &= \frac{1}{3}x + 1 \\
x - y &= 3
\end{align*} \]

you try...

\[ \begin{align*}
5x + 3y &= 65 \\
y - 25 &= -2x
\end{align*} \]

HW: Solving Linear Systems WS#2
Solving Linear Systems WS#2

Solve each system by graphing.

1) \(0 = -2 - y - x\)
   \(2y - x = 8\)

Solve the system by substitution.

2) \(y = x + 2\)
   \(y = 4x - 4\)

Solve each system by elimination.

3) \(12 - x = -4y\)
   \(12 - 9x = 12y\)

4) \(x = 1 + y\)
   \(0 = 8 - 4y + x\)

5) \(-1 + \frac{1}{4}x = \frac{1}{2}y\)
   \(-6x + 3y - 3 = 0\)

6) \(2 = -y + \frac{4}{3}x\)
   \(4x = 3y + 6\)
6.2 – Solving Linear-Quadratic Systems of Equations

2. **Linear-quadratic system**– A system of equations involving both a linear and quadratic equation involving the _______________________________. A graph of this system involves both a ____________________________ and a ____________________________.

The solution to a system of equations from a graph is the point(s) - or ordered pair(s) \((x, y)\) - where the two graphs ____________. Also called the _______________ points.

How many solutions are possible?

**Linear-quadratic system**

To solve a system of equations graphically:

1. _______________________________________________________________________
2. _______________________________________________________________________
3. _______________________________________________________________________

Solve the following system graphically.

a) \(3x + y = -9\)
\(0 = x^2 + 2x - y - 3\)
You try...

a) \[ 2x - y = -4 \]
\[ x + y + x^2 = 5x + 3 \]

6.2 – Solving Linear-Quadratic Systems of Equations (Part 2)

**Recall:** methods of **SOLVING** a quadratic equation:

1. Graphing
   i. Get equation into vertex form: \( y = a(x - p)^2 + q \), if necessary (use \( p \) and \( q \) formulas)
   ii. Plot the vertex \((p, q)\) follow the directions of the stretch factor ‘\(d\)’
   iii. Determine the coordinates where the parabolas meet

2. Factoring
   i. Combine both equations into one using **substitution** or **elimination**
   ii. Use decomposition (Rainbow Split) method to determine the factors of the quadratic
   iii. Set each factor equal to zero
   iv. Solve for (isolate) the variable

3. Quadratic Formula
   i. Combine both equations into one using **substitution** or **elimination**
   ii. Determine the values of \(a\), \(b\) and \(c\)
   iii. Substitute into the formula: 
      \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
      to determine the solution(s)

**HW:** Review Solving Quadratics WS
Review Solving Quadratics WS

Solve each equation by factoring.

1) \( x^2 - 2x - 15 = 0 \)

2) \( n^2 - 8n + 16 = 0 \)

3) \( x^2 + 2x + 2 = 5 \)

4) \( 6m^2 + 4m - 11 = 4 + 5m^2 + 2m \)

Solve each equation with the quadratic formula.

5) \( 2n^2 - n - 3 = 0 \)

6) \( 2a^2 - 3a - 2 = 0 \)

7) \( -7x^2 = 55 + x - 9x^2 \)

8) \( -p^2 - 8p - 13 = -10p^2 - 7 \)
You can also use **algebraic** methods to solve systems of equations.

Recall: steps to solving by **SUBSTITUTION**

Solve the following system of equations using substitution:

\[
\begin{align*}
5x - y &= 10 \\
a) \quad x^2 + x - 2y &= 0
\end{align*}
\]

You try...

\[
\begin{align*}
4x - y + 3 &= 0 \\
b) \quad 2x^2 + 8x - y + 3 &= 0
\end{align*}
\]
State if the point given is a solution to the system of equations.

1) \[2x^2 + 10x + 3y + 32 = 0\]
   \[-2x + y = 0\]
   Point: \((-4, -8)\)

2) \[-2x^2 - 35x + 2y - 165 = 0\]
   \[x + 2y = 3\]
   Point: \((-6, -9)\)

Solve each system of equations by substitution.

3) \[-2x^2 - 4x - y + 7 = 0\]
   \[-2x + y + 1 = 0\]
Solve each system of equations by graphing. Verify your solution.

4) \[ \begin{align*}
   y &= 2x^2 + 8x + 3 \\
   y &= 4x + 3
   \end{align*} \]

5) \[ \begin{align*}
   y + x^2 + 6x + 7 &= 0 \\
   y + x &= -3
   \end{align*} \]

Solve the system of equations by any method.

6) \[ \begin{align*}
   4x^2 - 3x - 12y + 43 &= 0 \\
   x + 2y &= 0
   \end{align*} \]
6.2 – Solving Linear-Quadratic Systems of Equations (Part 3)

You can also use algebraic methods to solve systems of equations.

Recall: steps to solving by **Elimination**

Solve the following system of equations

a) \( y + 2x = 3 \)
   \( y = x^2 - 6x + 3 \)

You try...

b) \( y = x^2 - 16x + 60 \)
   \( y = 12x - 55 \)

**HW**: System of Equations WS#4
Systems of Equations WS #4

State if the point given is a solution to the system of equations.

1) \( x^2 + 10x + y + 1 = 0 \)
   \( x^2 + 22x + y + 121 = 0 \)
   Point: \((-10, -1)\)

2) \(-x^2 + 3x + y + 5 = 0\)
   \(3x^2 - 9x + y - 11 = 0\)
   Point: \((-1, 4)\)

3) \(2x^2 + 12x + y + 17 = 0\)
   \(25x^2 + 58x + y + 40 = 0\)
   Point: \((-7, -3)\)

Solve each system of equations by elimination.

4) \(2x^2 - 4x + y = 3\)
   \(-2y + 4x = -7\)
2. **Quadratic-quadratic system** - A system of equations involving two quadratic equations involving the same variables. A graph of this system involves two ________________.

The **solution** to a system of equations from a graph is the point(s) - or ordered pair(s) \((x, y)\) - where the two graphs __________. Also called the _______________ points.

**How many solutions are possible?**

**Quadratic-quadratic system**

Solve the following system graphically.

a) \(y = x^2 + 2\)
   \(y = -x^2 + 2x + 6\)

You try...

b) \(y = -2(x + 4)^2 - 5\)
   \(y = x^2 + 8x + 8\)
You can also use algebraic methods to combine the equations. Then use FACTORING or QUADRATIC FORMULA to solve.

Solve the following system of equations using substitution:

\[ 3x^2 - 6x + 2 - 2y = 0 \]
\[ y = x^2 + x - 5 \]

You try...
\[ y = -2(x + 4)^2 - 5 \]
\[ y = -2x^2 - 16x - 37 = 0 \]

**HW:** System of Equations WS#5 questions #1-3
Solve the following system of equations using elimination:

\[
\begin{align*}
2x^2 + 12x - y + 18 &= 0 \\
b) -x^2 - 6x &= y - 3
\end{align*}
\]

You try...
\[
\begin{align*}
x^2 + 3x - y - 2 &= 0 \\
x^2 + y &= 4x - 3
\end{align*}
\]
Systems of Equations WS #5

Solve the system of equations by graphing. Verify your solution.

1) \[ g(x) = -x^2 + 2x + 6 \]
\[ f(x) = x^2 + 2 \]

Solve each system of equations by substitution. Verify your solution.

2) \[ x^2 - 8x + y - 152 = 0 \]
\[ 4x^2 + 46x + y + 91 = 0 \]

3) \[ x^2 + 7x + y + 12 = 0 \]
\[ 16x^2 + 112x + y + 162 = 0 \]

Solve each system of equations using elimination. Verify your solution.

4) \[ -x^2 - 4x + y + 2 = 0 \]
\[ x^2 - 4x + y + 2 = 0 \]

5) \[ -x^2 - 2x + y + 6 = 0 \]
\[ -8x^2 - 30x + y - 22 = 0 \]
Solve the system of equations by any method. Verify your solution.

6) \[-2x^2 - 14x + y - 15 = 0\]
   \[-18x^2 - 126x + y - 111 = 0\]

7) \[x^2 - 6x + y + 1 = 0\]
   \[12x^2 - 72x + y + 89 = 0\]
1. Solve graphically and verify your solution.
   \[ y = x^2 + 2x - 2 \]
   \[ y = -\frac{1}{4} x^2 - \frac{1}{2} x + \frac{7}{4} \]

2. Solve by \textbf{Substitution} and verify your solution.
   \[ 3x + y = -9 \]
   \[ 4x^2 - x + y = -9 \]

3. Solve by \textbf{Elimination} and verify your solution.
   \[ x + 2y = 46 \]
   \[ x^2 - 3y = 93 \]
4. Solve by the method of your choice and verify your solution.

\[ h = -4.9t^2 + 700 \]
\[ h = -5t + 650 \]

b) \[ 3x^2 + 4x - y - 8 = 0 \]
\[ y + 3 = 2x^2 + 4x \]