Discuss the Ideas

1. How do the area models and rectangle diagrams support the naming of a perfect square trinomial and a difference of squares binomial?
2. Why is it useful to identify the factoring patterns for perfect square trinomials and difference of squares binomials?
3. Why can you use the factors of a trinomial in one variable to factor a corresponding trinomial in two variables?

Exercises

A

4. Expand and simplify.

a) $(x + 2)^2$ **b)** $(3 - y)^2$
c) $(5 + d)^2$ **d)** $(7 - f)^2$
e) $(x + 2)(x - 2)$ **f)** $(3 - y)(3 + y)$
g) $(5 + d)(5 - d)$ **h)** $(7 - f)(7 + f)$

5. Identify each polynomial as a perfect square trinomial, a difference of squares, or neither.

a) $25 - t^2$
b) $16m^2 + 49n^2$
c) $4x^2 - 24xy + 9y^2$
d) $9m^2 - 24mn + 16n^2$

6. Factor each binomial.

a) $x^2 - 49$ **b)** $b^2 - 121$
c) $1 - q^2$ **d)** $36 - c^2$

B

7. **a)** Factor each trinomial.

i) $a^2 + 10a + 25$ **ii)** $b^2 - 12b + 36$
iii) $c^2 + 14c + 49$ **iv)** $d^2 - 16d + 64$
v) $e^2 + 18e + 81$ **vi)** $f^2 - 20f + 100$

b) What patterns do you see in the trinomials and their factors in part a? Write the next 4 trinomials in the pattern and their factors.

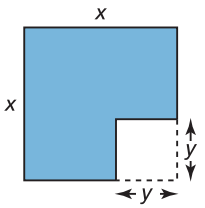
8. Factor each trinomial. Verify by multiplying the factors.

a) $4x^2 - 12x + 9$ **b)** $9 + 30n + 25n^2$
c) $81 - 36v + 4v^2$ **d)** $25 + 40h + 16h^2$
e) $9g^2 + 48g + 64$ **f)** $49r^2 - 28r + 4$

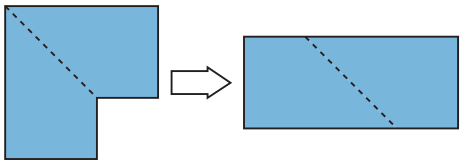
9. **a)** Cut out a square from a piece of paper. Let x represent the side length of the square. Write an expression for the area of the square.

Cut a smaller square from one corner. Let y represent the side length of the cut-out square. Write an expression for the area of the cut-out square.

Write an expression for the area of the piece that remains.



b) Cut the L-shaped piece into 2 congruent pieces, then arrange as shown below.



What are the dimensions of this rectangle, in terms of x and y?
What is the area of this rectangle?

c) Explain how the results of parts a and b illustrate the difference of squares.

10. Factor each binomial. Verify by multiplying the factors.

a) $9d^2 - 16f^2$ **b)** $25s^2 - 64t^2$

c) $144a^2 - 9b^2$ **d)** $121m^2 - n^2$

e) $81k^2 - 49m^2$ **f)** $100y^2 - 81z^2$

g) $v^2 - 36t^2$ **h)** $4j^2 - 225h^2$

11. Factor each trinomial.

a) $y^2 + 7yz + 10z^2$ **b)** $4w^2 - 8wx - 21x^2$

c) $12s^2 - 7su + u^2$ **d)** $3t^2 - 7tv + 4v^2$

e) $10r^2 + 9rs - 9s^2$ **f)** $8p^2 + 18pq - 35q^2$

12. Factor each trinomial. Which trinomials are perfect squares?

a) $4x^2 + 28xy + 49y^2$ **b)** $15m^2 + 7mn - 4n^2$

c) $16r^2 + 8rt + t^2$ **d)** $9a^2 - 42ab + 49b^2$

e) $12h^2 + 25hk + 12k^2$ **f)** $15f^2 - 31fg + 10g^2$

13. Factor.

a) $8m^2 - 72n^2$ **b)** $8z^2 + 8yz + 2y^2$

c) $12x^2 - 27y^2$ **d)** $8p^2 + 40pq + 50q^2$

e) $-24u^2 - 6uv + 9v^2$ **f)** $-18b^2 + 128c^2$

14. A circular fountain has a radius of r centimetres. It is surrounded by a circular flower bed with radius R centimetres.

a) Sketch and label a diagram.

b) How can you use the difference of squares to determine an expression for the area of the flower bed?

c) Use your expression from part b to calculate the area of the flower bed when $r = 150$ cm and $R = 350$ cm.

15. **a)** Find an integer to replace $\square$ so that each trinomial is a perfect square.

i) $x^2 + \square x + 49$

ii) $4a^2 + 20ab + \square b^2$

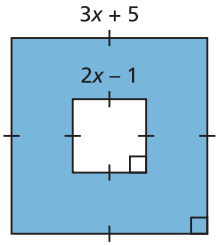
iii) $\square c^2 - 24cd + 16d^2$

b) How many integers are possible for each trinomial in part a? Explain why no more integers are possible.

16. Find consecutive integers a, b, and c so that the trinomial $ax^2 + bx + c$ can be factored. How many possibilities can you find?

17. Use mental math to determine $(199)(201)$. Explain your strategy.

18. Determine the area of the shaded region. Simplify your answer.



C

19. **a)** Identify each expression as a perfect square trinomial, a difference of squares, or neither. Justify your answers.

i) $(x^2 + 5)^2$

ii) $-100 + r^2$

iii) $81a^2b^2 - 1$

iv) $16s^4 + 8s^2 + 1$

b) Which expressions in part a can be factored? Factor each expression you identify.

20. Factor fully.

a) $x^4 - 13x^2 + 36$

b) $a^4 - 17a^2 + 16$

c) $y^4 - 5y^2 + 4$

21. Factor, if possible. For each binomial that cannot be factored, explain why.

a) $8d^2 - 32e^2$

b) $25m^2 - \frac{1}{4}n^2$

c) $18x^2y^2 - 50y^4$

d) $25s^2 + 49t^2$

e) $10a^2 - 7b^2$

f) $\frac{x^2}{16} - \frac{y^2}{49}$

Reflect

Explain how a difference of squares binomial is a special case of a trinomial. How is factoring a difference of squares like factoring a trinomial? How is it different? Include examples in your explanation.