

Assess Your Understanding

- Give 4 examples of radicals. Use a different index for each radical.
 - Identify the radicand and index for each radical.
 - Explain the meaning of the index of each radical.
- Evaluate each radical. Justify your answer.
 - $\sqrt{36}$
 - $\sqrt[3]{8}$
 - $\sqrt[4]{10\,000}$
 - $\sqrt[5]{-32}$
 - $\sqrt[3]{\frac{27}{125}}$
 - $\sqrt{2.25}$
 - $\sqrt[3]{0.125}$
 - $\sqrt[4]{625}$
- Estimate the value of each radical to 1 decimal place.
What strategy did you use?
 - $\sqrt{8}$
 - $\sqrt[3]{9}$
 - $\sqrt[4]{10}$
 - $\sqrt{13}$
 - $\sqrt[3]{15}$
 - $\sqrt[4]{17}$
 - $\sqrt{19}$
 - $\sqrt[3]{20}$
- What happens when you attempt to determine the square root of a number such as -4 ? Explain the result.
 - For which other radical indices do you get the same result with a negative radicand, as in part a?
 - When a radicand is negative:
 - Which types of radicals can be evaluated or estimated?
 - Which types of radicals cannot be evaluated or estimated?
- For each number below, write an equivalent form as:
 - a square root
 - a cube root
 - a fourth root
 - 2
 - 3
 - 4
 - 10
 - 0.9
 - 0.2
- Choose values of n and x so that $\sqrt[n]{x}$ is:
 - a whole number
 - a negative integer
 - a rational number
 - an approximate decimal

Verify your answers.

$\sqrt[3]{7} = 1.912\,931\,182\,772\,389\,101\,199\,116\,839\,548\,760\,282\,862\,439\,050\,345\,875\,766\,210\,647\,640\,447\,234\,276\,179\,230\,756\,007\,525\,441\,477\,285\,709\,904\,541\,913\,958\,790\,759\,227\,944\,615\,293\,864\,212\,013\,147\,486\,695\,712\,445\,614\,039\,888\,169\,681\,471\,379\,702\,626\,745\,446\,612\,044\,061\,147\,761\,416\,391\,806\,241\,578\,673\,927\,453\,141\,892\,781\,075\,667\,871\,691\,066\,794\,229\,608\,191\,383\,758\,219\,601\,042\,802\,155\,946\,150\,300\,697\,613\,551\,307\,287\,191\,167\,449\,608\,313\,771\,081\,504\,584\,906\,733\,629\,612\,655\,131\,887\,183\,073\,974\,740\,458\,182\,893\,551\,185\,633\,773\,547\,212\,430\,828\,593\,092\,438\,654\,681\,098\,440\,938\,923\,431\,110\,568\,208\,310\,066\,222\,313\,508\,685\,604\,140\,201\,133\,691\,676\,872\,961\,909\,991\,081\,229\,243\,112\,174\,410\,739\,919\,535\,437\,911\,589\,068\,649\,306\,417\,647\,062\,891\,485\,738\,710\,386\,488\,768\,546\,101\,412\,787\,971\,783\,309\,636\,271\,779\,870\,721\,786\, \dots$

Discuss the Ideas

- How do you determine whether a radical represents a rational or an irrational number? Use examples to explain.
- How can you determine whether the decimal form of a radical represents its exact value?

Exercises

A

3. Tell whether each number is rational or irrational.

- a) $\sqrt{12}$ b) $\sqrt[4]{16}$
 c) $\sqrt[3]{-100}$ d) $\sqrt{\frac{4}{9}}$
 e) $\sqrt{1.25}$ f) 1.25

4. Classify each number below as:

- a) a natural number b) an integer
 c) a rational number d) an irrational number

$\frac{4}{3}$, $0.3\bar{4}$, -5 , $\sqrt[4]{9}$, -2.1538 , $\sqrt[3]{27}$, 7

B

5. a) Why are $\sqrt{49}$ and $\sqrt[4]{16}$ rational numbers?
 b) Why are $\sqrt{21}$ and $\sqrt[3]{36}$ irrational numbers?

6. Look at this calculator screen.

$\sqrt{(150)}$
 12.24744871

- a) Is the number 12.247 448 71 rational or irrational? Explain.
 b) Is the number $\sqrt{150}$ rational or irrational? Explain.
7. a) Sketch a diagram to represent the set of rational numbers and the set of irrational numbers.
 b) Write each number that follows in the correct set.
 $\frac{1}{2}$, $-\sqrt{3}$, $\sqrt{4}$, $\sqrt[4]{5}$, $-\frac{7}{6}$, $\sqrt[3]{8}$, 10.12, $-13.\bar{4}$,
 $\sqrt{0.15}$, $\sqrt{0.16}$, 17
8. For which numbers will the cube root be irrational? Use 2 different strategies to justify your answers.
 a) 8 b) 64 c) 30 d) 300

9. Sketch a number line for each irrational number and label its approximate location. Explain your reasoning.

- a) $\sqrt{5}$ b) $\sqrt[3]{12}$ c) $\sqrt[4]{25}$ d) $\sqrt[3]{-12}$

10. Use a number line to order the irrational numbers in each set from greatest to least.

- a) $\sqrt[3]{70}$, $\sqrt{50}$, $\sqrt[4]{100}$, $\sqrt[3]{400}$
 b) $\sqrt{89}$, $\sqrt[4]{250}$, $\sqrt[3]{-150}$, $\sqrt[3]{150}$

11. Use a number line to order these numbers from least to greatest. How can you verify your answer?

$\sqrt{40}$, $\sqrt[3]{500}$, $\sqrt{98}$, $\sqrt[3]{98}$, $\sqrt{75}$, $\sqrt[3]{300}$

12. Use a number line to order these numbers from least to greatest. Identify which numbers are irrational and which are rational.

$-\frac{14}{5}$, $\frac{123}{99}$, -2 , $\sqrt[3]{-10}$, $\sqrt{4}$

13. How do you use irrational numbers when you calculate the length of the hypotenuse of a right triangle with legs 5 cm and 3 cm?

14. a) Which of the following statements are true? Explain your reasoning.

- i) All natural numbers are integers.
 ii) All integers are rational numbers.
 iii) All whole numbers are natural numbers.
 iv) All irrational numbers are roots.
 v) Some rational numbers are natural numbers.

- b) For each statement in part a that is false, provide examples to explain why.

15. Write a number that is:

- a) a rational number but not an integer
 b) a whole number but not a natural number
 c) an irrational number