Discuss the Ideas

1. When a is a rational number and n is a natural number, what does $a^{\frac{1}{n}}$ represent?

2. When a is a rational number and m and n are natural numbers, what does $a^{\frac{m}{n}}$ represent?

Exercises

A

3. Evaluate each power without using a calculator.

a) $16^{\frac{1}{2}}$ **b)** $36^{\frac{1}{2}}$ **c)** $64^{\frac{1}{3}}$

d) $32^{\frac{1}{5}}$ **e)** $(-27)^{\frac{1}{3}}$ **f)** $(-1000)^{\frac{1}{3}}$

4. Evaluate each power without using a calculator.

a) $100^{0.5}$ **b)** $81^{0.25}$

c) $1024^{0.2}$ **d)** $(-32)^{0.2}$

5. Write each power as a radical.

a) $36^{\frac{1}{3}}$ **b)** $48^{\frac{1}{2}}$ **c)** $(-30)^{\frac{1}{5}}$

6. Write each radical as a power.

a) $\sqrt{39}$ **b)** $\sqrt[4]{90}$

c) $\sqrt[3]{29}$ **d)** $\sqrt[5]{100}$

7. Evaluate each power without using a calculator.

a) 8^{0} **b)** $8^{\frac{1}{3}}$ **c)** $8^{\frac{2}{3}}$

d) $8^{\frac{3}{3}}$ **e)** $8^{\frac{4}{3}}$ **f)** $8^{\frac{5}{3}}$

B

8. Write each power as a radical.

a) $4^{\frac{2}{3}}$ **b)** $(-10)^{\frac{3}{5}}$ **c)** $2.3^{\frac{3}{2}}$

9. A cube has a volume of 350 cm^3. Write the edge length of the cube as a radical and as a power.

10. Write each power as a radical.

a) $48^{\frac{2}{3}}$ **b)** $(-1.8)^{\frac{5}{3}}$ **c)** $\left(\frac{3}{8}\right)^{2.5}$

d) $0.75^{0.75}$ **e)** $\left(-\frac{5}{9}\right)^{\frac{2}{5}}$ **f)** $1.25^{1.5}$

11. Write each radical as a power.

a) $\sqrt{3.8^3}$ **b)** $\left(\sqrt[3]{-1.5}\right)^2$ **c)** $\sqrt[4]{\left(\frac{9}{5}\right)^5}$

d) $\sqrt[3]{\left(\frac{3}{8}\right)^4}$ **e)** $\left(\sqrt{\frac{5}{4}}\right)^3$ **f)** $\sqrt[5]{(-2.5)^3}$

12. Evaluate each power without using a calculator.

a) $9^{\frac{3}{2}}$ **b)** $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **c)** $(-27)^{\frac{2}{3}}$

d) $0.36^{1.5}$ **e)** $(-64)^{\frac{2}{3}}$ **f)** $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

13. Write an equivalent form for each number using a power with exponent $\frac{1}{2}$, then write the answer as a radical.

a) 2 **b)** 4 **c)** 10 **d)** 3 **e)** 5

14. Write an equivalent form for each number using a power with exponent $\frac{1}{3}$, then write the answer as a radical.

a) -1 **b)** 2 **c)** 3 **d)** -4 **e)** 4

15. Arrange these numbers in order from least to greatest. Describe your strategy.

$\sqrt[3]{4}$, $4^{\frac{3}{2}}$, 4^2, $\left(\frac{1}{4}\right)^{\frac{3}{2}}$

16. **a)** Evaluate.

i) $16^{1.5}$ **ii)** $81^{0.75}$

iii) $(-32)^{0.8}$ **iv)** $35^{0.5}$

v) $1.21^{1.5}$ **vi)** $\left(\frac{3}{4}\right)^{0.6}$

b) Which powers in part a could you have evaluated without a calculator? How can you tell before you evaluate?

17. The height, h metres, of a certain species of fir tree can be estimated from the formula $h = 35d^{\frac{2}{3}}$, where d metres is the diameter at the base. Use the formula to determine the approximate height of a fir tree with base diameter 3.2 m.

18. Here is a student's solution for evaluating a power.

$$1.96^{\frac{3}{2}} = \left(\sqrt[3]{1.96}\right)^2$$
$$= (1.2514\ldots)^2$$
$$= 1.5661\ldots$$

Identify the errors the student made. Write a correct solution.

19. A formula for the approximate surface area, SA square metres, of a person's body is $SA = 0.096m^{0.7}$, where m is the person's mass in kilograms. Calculate the surface area of a child with mass 40 kg.

20. Here is an expression for the percent of caffeine that remains in your body n hours after you drink a caffeine beverage:

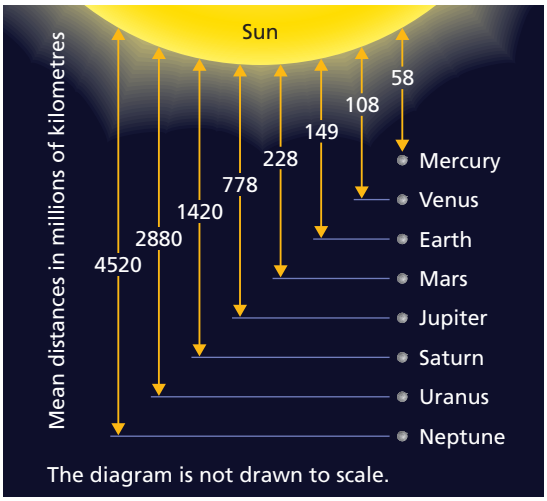
$100(0.5)^{\frac{n}{5}}$

a) Show that this expression and the expression on page 222 give the same result, to the nearest whole number, for the percent of caffeine that remains after $\frac{1}{2}$ h.

b) Use the expression above to determine the percent of caffeine that remains after 1.5 h.

c) After how many hours does 50% of the caffeine remain? Explain how you know.

21. In the late 1500s, Johannes Kepler developed a formula to calculate the time it takes each planet to orbit the sun (called the *period*). The formula is $T \doteq 0.2R^{\frac{3}{2}}$, where T is the period in Earth days and R is the mean distance from the planet to the sun in millions of kilometres.



The mean distance of Earth from the sun is about 149 million kilometres. The mean distance of Mars from the sun is about 228 million kilometres. Which planet has the longer period, Earth or Mars? Justify your answer.

C

22. Two students discussed the meaning of the statement $3.2^{4.2} = 132.3213...$

Luc said: It means 3.2 multiplied by itself 4.2 times is about 132.3213.

Karen said: No, you can't multiply a number 4.2 times. $3.2^{4.2}$ can be written as $3.2^{\frac{42}{10}}$. So the statement means that 42 factors, each equal to the tenth root of 3.2, multiplied together will equal about 132.3213.

Which student is correct? Explain.

Reflect

In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

What must be true about x for $x^{\frac{m}{n}}$ to be a rational number?