

The six parts of the triangle are as follows:

$$\angle A = 144.8^\circ \quad a = 11$$

$$\angle B = 15.2^\circ \quad b = 5$$

$$\angle C = 20^\circ \quad c = 6.5$$

Your Turn

In $\triangle ABC$, $a = 9$, $b = 7$, and $\angle C = 33.6^\circ$. Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles, to the nearest tenth.

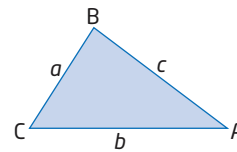
Key Ideas

- Use the cosine law to find the length of an unknown side of any triangle when you know the lengths of two sides and the measure of the angle between them.
- The cosine law states that for any $\triangle ABC$, the following relationships exist:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$
- Use the cosine law to find the measure of an unknown angle of any triangle when the lengths of three sides are known.
- Rearrange the cosine law to solve for a particular angle.
For example, $\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.
- Use the cosine law in conjunction with the sine law to solve a triangle.

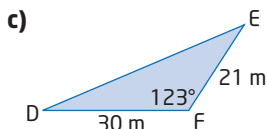
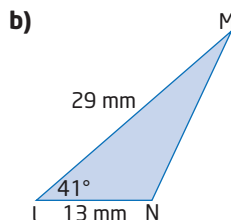
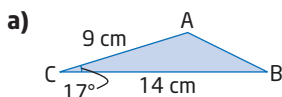


Check Your Understanding

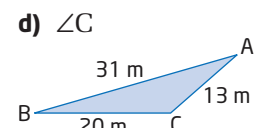
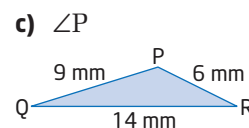
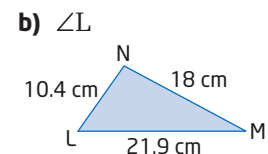
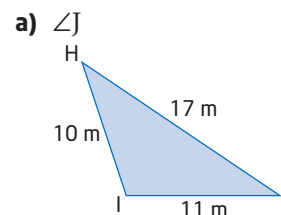
Practise

Where necessary, round lengths to the nearest tenth of a unit and angles to the nearest degree, unless otherwise stated.

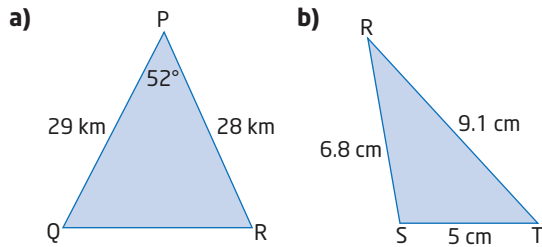
1. Determine the length of the third side of each triangle.



2. Determine the measure of the indicated angle.



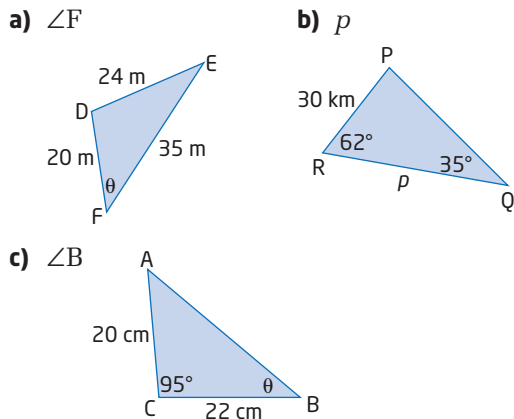
3. Determine the lengths of the unknown sides and the measures of the unknown angles.



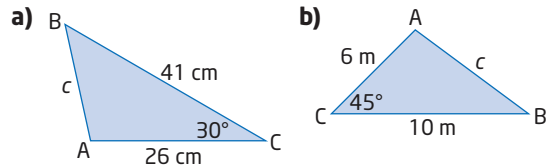
4. Make a sketch to show the given information for each $\triangle ABC$. Then, determine the indicated value.
- $AB = 24$ cm, $AC = 34$ cm, and $\angle A = 67^\circ$. Determine the length of BC .
 - $AB = 15$ m, $BC = 8$ m, and $\angle B = 24^\circ$. Determine the length of AC .
 - $AC = 10$ cm, $BC = 9$ cm, and $\angle C = 48^\circ$. Determine the length of AB .
 - $AB = 9$ m, $AC = 12$ m, and $BC = 15$ m. Determine the measure of $\angle B$.
 - $AB = 18.4$ m, $BC = 9.6$ m, and $AC = 10.8$ m. Determine the measure of $\angle A$.
 - $AB = 4.6$ m, $BC = 3.2$ m, and $AC = 2.5$ m. Determine the measure of $\angle C$.

Apply

5. Would you use the sine law or the cosine law to determine each indicated side length or angle measure? Give reasons for your choice.



6. Determine the length of side c in each $\triangle ABC$, to the nearest tenth.



7. In a parallelogram, the measure of the obtuse angle is 116° . The adjacent sides, containing the angle, measure 40 cm and 22 cm, respectively. Determine the length of the longest diagonal.
8. The longest tunnel in North America could be built through the mountains of the Kicking Horse Canyon, near Golden, British Columbia. The tunnel would be on the Trans-Canada highway connecting the Prairies with the west coast. Suppose the surveying team selected a point A, 3000 m away from the proposed tunnel entrance and 2000 m from the tunnel exit. If $\angle A$ is measured as 67.7° , determine the length of the tunnel, to the nearest metre.

Web Link

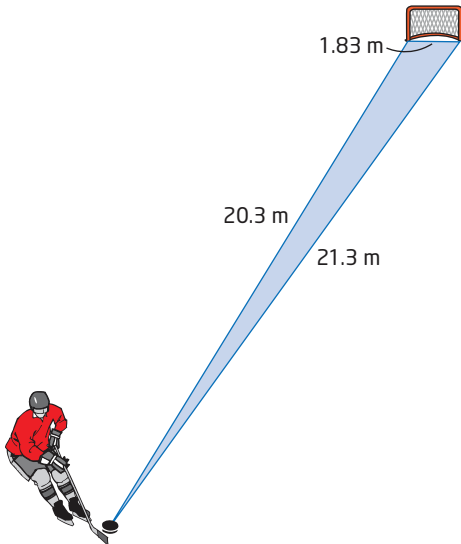
To learn more about the history of the Kicking Horse Pass and proposed plans for a new tunnel, go to www.mhrprecalc1.ca and follow the links.

9. Thousands of Canadians are active in sailing clubs. In the Paralympic Games, there are competitions in the single-handed, double-handed, and three-person categories. A sailing race course often follows a triangular route around three buoys. Suppose the distances between the buoys of a triangular course are 8.56 km, 5.93 km, and 10.24 km. Determine the measure of the angle at each of the buoys.

Did You Know?

Single-handed sailing means that one person sails the boat. Double-handed refers to two people. Buoys are floating markers, anchored to keep them in place. The oldest record of buoys being used to warn of rock hazards is in the thirteenth century on the Guadalquivir River near Seville in Spain.

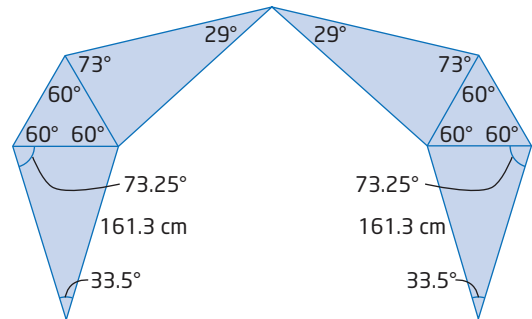
- 10.** The Canadian women's national ice hockey team has won numerous international competitions, including gold medals at the 2002, 2006, and 2010 Winter Olympics. A player on the blue line shoots a puck toward the 1.83-m-wide net from a point 20.3 m from one goal post and 21.3 m from the other. Within what angle must she shoot to hit the net? Answer to the nearest tenth of a degree.



- 11.** One of the best populated sea-run brook trout areas in Canada is Lac Guillaume-Delisle in northern Québec. Also known as Richmond Gulf, it is a large triangular-shaped lake. Suppose the sides forming the northern tip of the lake are 65 km and 85 km in length, and the angle at the northern tip is 7.8° . Determine the width of the lake at its base.



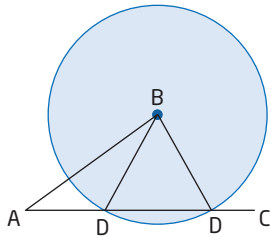
- 12.** An aircraft-tracking station determines the distances from a helicopter to two aircraft as 50 km and 72 km. The angle between these two distances is 49° . Determine the distance between the two aircraft.
- 13.** Tony Smith was born in 1912 in South Orange, New Jersey. As a child, Tony suffered from tuberculosis. He spent his time playing and creating with medicine boxes. His sculpture, Moondog, consists of several equilateral and isosceles triangles that combine art and math. Use the information provided on the diagram to determine the maximum width of Moondog.



Moondog by Tony Smith

- 14.** Julia and Isaac are backpacking in Banff National Park. They walk 8 km from their base camp heading $N42^\circ E$. After lunch, they change direction to a bearing of 137° and walk another 5 km.
- Sketch a diagram of their route.
 - How far are Julia and Isaac from their base camp?
 - At what bearing must they travel to return to their base camp?

Step 4



- a) Yes.
 b) Two triangles can be formed when BD is greater than the distance from B to the line AC .

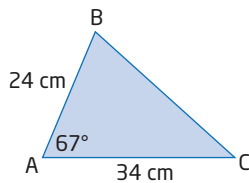
Step 5

- a) Yes.
 b) One triangle is formed when BD is greater than the length AB .

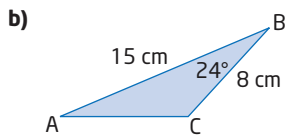
Step 6 The conjectures will work so long as $\angle A$ is an acute angle. The relationship changes when $\angle A > 90^\circ$.

2.4 The Cosine Law, pages 119 to 125

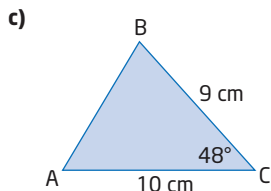
1. a) 6.0 cm b) 21.0 mm c) 45.0 m
 2. a) $\angle J = 34^\circ$ b) $\angle L = 55^\circ$
 c) $\angle P = 137^\circ$ d) $\angle C = 139^\circ$
 3. a) $\angle Q = 62^\circ$, $\angle R = 66^\circ$, $p = 25.0$ km
 b) $\angle S = 100^\circ$, $\angle R = 33^\circ$, $\angle T = 47^\circ$
 4. a)



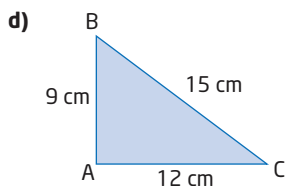
$BC = 33.1$ cm



$AC = 8.4$ m

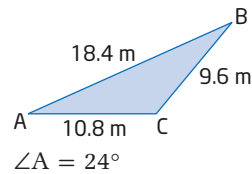


$AB = 7.8$ cm

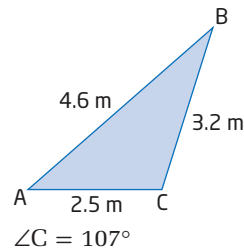


$\angle B = 53^\circ$

e)

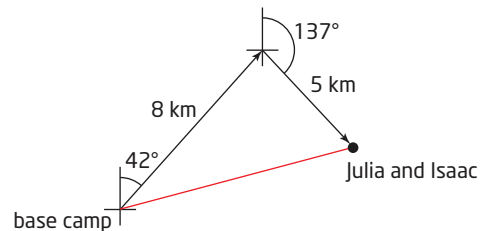


f)



5. a) Use the cosine law because three sides are given (SSS). There is no given angle and opposite side to be able to use the sine law.
 b) Use the sine law because two angles and an opposite side are given.
 c) Use the cosine law to find the missing side length. Then, use the sine law to find the indicated angle.

6. a) 22.6 cm
 b) 7.2 m
 7. 53.4 cm
 8. 2906 m
 9. The angles between the buoys are 35° , 88° , and 57° .
 10. 4.2°
 11. 22.4 km
 12. 54.4 km
 13. 458.5 cm
 14. a)



- b) 9.1 km
 c) 255°
 15. 9.7 m
 16. Use the cosine law in each oblique triangle to find the measure of each obtuse angle. These three angles meet at a point and should sum to 360° . The three angles are 118° , 143° , and 99° . Since $118^\circ + 143^\circ + 99^\circ = 360^\circ$, the side measures are accurate.
 17. The interior angles of the bike frame are 73° , 62° , and 45° .
 18. 98.48 m
 19. 1546 km