Check Your Understanding

Practise

1. Simplify each product. Identify all non-permissible values.

a) $\frac{12m^2f}{5cf} \times \frac{15c}{4m}$

b) $\frac{3(a-b)}{(a-1)(a+5)} \times \frac{(a-5)(a+5)}{15(a-b)}$

c) $\frac{(y-7)(y+3)}{(2y-3)(2y+3)} \times \frac{4(2y+3)}{(y+3)(y-1)}$

2. Write each product in simplest form. Determine all non-permissible values.

a) $\frac{d^2-100}{144} \times \frac{36}{d+10}$

b) $\frac{a+3}{a+1} \times \frac{a^2-1}{a^2-9}$

c) $\frac{4z^2-25}{2z^2-13z+20} \times \frac{z-4}{4z+10}$

d) $\frac{2p^2+5p-3}{2p-3} \times \frac{p^2-1}{6p-3} \times \frac{2p-3}{p^2+2p-3}$

3. What is the reciprocal of each rational expression?

a) $\frac{2}{t}$ **b)** $\frac{2x-1}{3}$

c) $\frac{-8}{3-y}$ **d)** $\frac{2p-3}{p-3}$

4. What are the non-permissible values in each quotient?

a) $\frac{4t^2}{3s} \div \frac{2t}{s^2}$

b) $\frac{r^2-7r}{r^2-49} \div \frac{3r^2}{r+7}$

c) $\frac{5}{n+1} \div \frac{10}{n^2-1} \div (n-1)$

5. What is the simplified product of $\frac{2x-6}{x+3}$ and $\frac{x+3}{2}$? Identify any non-permissible values.

6. What is the simplified quotient of $\frac{y^2}{y^2-9}$ and $\frac{y}{y-3}$? Identify any non-permissible values.

7. Show how to simplify each rational expression or product.

a) $\frac{3-p}{p-3}$

b) $\frac{7k-1}{3k} \times \frac{1}{1-7k}$

8. Express each quotient in simplest form. Identify all non-permissible values.

a) $\frac{2w^2-w-6}{3w+6} \div \frac{2w+3}{w+2}$

b) $\frac{v-5}{v} \div \frac{v^2-2v-15}{v^3}$

c) $\frac{9x^2-1}{x+5} \div \frac{3x^2-5x-2}{2-x}$

d) $\frac{8y^2-2y-3}{y^2-1} \div \frac{2y^2-3y-2}{2y-2} \div \frac{3-4y}{y+1}$

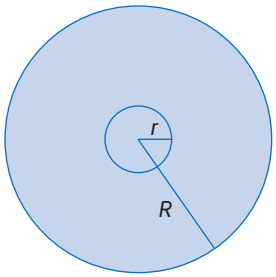
9. Explain why the non-permissible values in the quotient $\frac{x-5}{x+3} \div \frac{x+1}{x-2}$ are -3, -1 and 2.

Apply

10. The height of a stack of plywood is represented by $\frac{n^2-4}{n+1}$. If the number of sheets is defined by $n-2$, what expression could be used to represent the thickness of one sheet? Express your answer in simplest form.

28. a) Lt

b)



$\pi(R - r)(R + r)$

c) $L = \dfrac{\pi(R + r)(R - r)}{t}$, $t > 0$, $R > r$, and t, R, and r should be expressed in the same units.

29. Examples:

a) $\dfrac{2}{(x + 2)(x - 5)}$

b) $\dfrac{x^2 + 3x}{x^2 + 2x - 3}$; the given expression has a non-permissible value of -1. Multiply the numerator and denominator by a factor, $x + 3$, that has a non-permissible value of -3.

30. a) Example: if $y = 7$,

$\dfrac{y - 3}{4} = \dfrac{7 - 3}{4} = 1$ and $\dfrac{2y^2 - 5y - 3}{8y + 4} = \dfrac{2(7^2) - 5(7) - 3}{8(7) + 4} = \dfrac{60}{60} = 1$

b) $\dfrac{2y^2 - 5y - 3}{8y + 4} = \dfrac{(2y + 1)(y - 3)}{4(2y + 1)} = \dfrac{y - 3}{4}$

c) The algebraic approach, in part b), proves that the expressions are equivalent for all values of y, except the non-permissible value.

31. a) $m = \dfrac{p - 8}{p + 1}$

b) Any value $-1 < p < 8$ will give a negative slope. Example: If $p = 0$, $m = \dfrac{-8}{1}$.

c) If $p = -1$, then the expression is undefined, and the line is vertical.

32. Example: $\dfrac{12}{15} = \dfrac{(3)(4)}{(3)(5)} = \dfrac{4}{5}$,

$\dfrac{x^2 - 4}{x^2 + 5x + 6} = \dfrac{(x + 2)(x - 2)}{(x + 3)(x + 2)} = \dfrac{(x - 2)}{(x + 3)}$, $x \neq -3, -2$

6.2 Multiplying and Dividing Rational Expressions, pages 327 to 330

1. a) $9m$, $c \neq 0$, $f \neq 0$, $m \neq 0$

b) $\dfrac{a - 5}{5(a - 1)}$, $a \neq -5$, 1, $a \neq b$

c) $\dfrac{4(y - 7)}{(2y - 3)(y - 1)}$, $y \neq -3$, 1, $\pm\dfrac{3}{2}$

2. a) $\dfrac{d - 10}{4}$, $d \neq -10$ **b)** $\dfrac{a - 1}{a - 3}$, $a \neq \pm 3$, -1

c) $\dfrac{1}{2}$, $z \neq 4$, $\pm\dfrac{5}{2}$

d) $\dfrac{p + 1}{3}$, $p \neq -3$, 1, $\dfrac{3}{2}$, $\dfrac{1}{2}$

3. a) $\dfrac{t}{2}$ **b)** $\dfrac{3}{2x - 1}$

c) $\dfrac{y - 3}{8}$ **d)** $\dfrac{p - 3}{2p - 3}$

4. a) $s \neq 0$, $t \neq 0$ **b)** $r \neq \pm 7$, 0

c) $n \neq \pm 1$

5. $x - 3$, $x \neq -3$

6. $\dfrac{y}{y + 3}$, $y \neq \pm 3$, 0

7. a) $\dfrac{3 - p}{p - 3} = \dfrac{-1(p - 3)}{p - 3} = -1$, $p \neq 3$

b) $\dfrac{7k - 1}{3k} \times \dfrac{1}{1 - 7k} = \dfrac{7k - 1}{3k} \times \dfrac{1}{-1(7k - 1)} = \dfrac{-1}{3k}$ or $-\dfrac{1}{3k}$, $k \neq 0$, $\dfrac{1}{7}$

8. a) $\dfrac{w - 2}{3}$, $w \neq -2$, $-\dfrac{3}{2}$

b) $\dfrac{v^2}{v + 3}$, $v \neq 0$, -3, 5,

c) $\dfrac{-1(3x - 1)}{x + 5}$, $x \neq -5$, 2, $-\dfrac{1}{3}$

d) $\dfrac{-2}{y - 2}$, $y \neq \pm 1$, 2, $-\dfrac{1}{2}$, $\dfrac{3}{4}$

9. -3 and -2 are the non-permissible values of the original denominators, and -1 is the non-permissible value when the reciprocal of the divisor is created.

10. $\dfrac{n^2 - 4}{n + 1} \div (n - 2)$; $\dfrac{n + 2}{n + 1}$, $n \neq -1$, 2

11. a) $\dfrac{(x - 3)}{5}(60) = 12x - 36$ metres

b) $900 \div \dfrac{600}{n + 1} = \dfrac{3n + 3}{2}$ kilometres per hour, $n \neq -1$

c) $\dfrac{x^2 + 2x + 1}{(2x - 3)(x + 1)} = \dfrac{x + 1}{2x - 3}$ metres, $x \neq \dfrac{3}{2}$, -1

12. They are reciprocals of each other. This is always true. The divisor and dividend are interchanged.

13. Example:

$1 \text{ yd}\left(\dfrac{3 \text{ ft}}{1 \text{ yd}}\right)\left(\dfrac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\dfrac{2.54 \text{ cm}}{1 \text{ in.}}\right) = 91.44 \text{ cm}$

14. a) Tessa took the reciprocal of the dividend, not the divisor.

b) $= \dfrac{(c + 6)(c - 6)}{2c} \times \dfrac{8c^2}{c + 6}$

$= 4c(c - 6)$

$= 4c^2 - 24c$, $c \neq 0, -6$