

Key Ideas

- You can solve a rational equation by multiplying both sides by a common denominator. This eliminates the fractions from the equation. Then, solve the resulting equation.
- When solving a word problem involving rates, it is helpful to use a table.
- Check that the potential roots satisfy the original equation, are not non-permissible values, and, in the case of a word problem, are realistic in the context.

Check Your Understanding

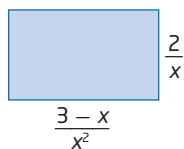
Practise

- Use the LCD to eliminate the fractions from each equation. Do not solve.
 - $\frac{x-1}{3} - \frac{2x-5}{4} = \frac{5}{12} + \frac{x}{6}$
 - $\frac{2x+3}{x+5} + \frac{1}{2} = \frac{7}{2x+10}$
 - $\frac{4x}{x^2-9} - \frac{5}{x+3} = 2$
- Solve and check each equation. Identify all non-permissible values.
 - $\frac{f+3}{2} - \frac{f-2}{3} = 2$
 - $\frac{3-y}{3y} + \frac{1}{4} = \frac{1}{2y}$
 - $\frac{9}{w-3} - \frac{4}{w-6} = \frac{18}{w^2-9w+18}$
- Solve each rational equation. Identify all non-permissible values.
 - $\frac{6}{t} + \frac{t}{2} = 4$
 - $\frac{6}{c-3} = \frac{c+3}{c^2-9} - 5$
 - $\frac{d}{d+4} = \frac{2-d}{d^2+3d-4} + \frac{1}{d-1}$
 - $\frac{x^2+x+2}{x+1} - x = \frac{x^2-5}{x^2-1}$
- Joline solved the following rational equation. She claims that the solution is $y = 1$. Do you agree? Explain.

$$\frac{-3y}{y-1} + 6 = \frac{6y-9}{y-1}$$

Apply

- A rectangle has the dimensions shown.



- What is an expression for the difference between the length and the width of the rectangle? Simplify your answer.
 - What is an expression for the area of the rectangle? Express the answer in simplest form.
 - If the perimeter of the rectangle is 28 cm, find the value(s) for x .
- Solve. Round answers to the nearest hundredth.
 - $\frac{26}{b+5} = 1 + \frac{3}{b-2}$
 - $\frac{c}{c+2} - 3 = \frac{-6}{c^2-4}$
 - Experts claim that the golden rectangle is most pleasing to the eye. It has dimensions that satisfy the equation $\frac{l}{w} = \frac{l+w}{l}$, where w is the width and l is the length. According to this relationship, how long should a rectangular picture frame be if its width is 30 cm? Give the exact answer and an approximate answer, rounded to the nearest tenth of a centimetre.
 - The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers.

23. 3

24. Examples: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$ and

$$\frac{2}{5} + \frac{1}{3} = \frac{2(3) + 1(5)}{15} = \frac{11}{15}$$

$$\frac{2}{x} + \frac{1}{x} = \frac{2+1}{x} = \frac{3}{x} \text{ and}$$

$$\frac{2}{x} + \frac{1}{y} = \frac{2(y) + 1(x)}{xy} = \frac{2y+x}{xy}$$

25. a) The student's suggestion is correct.

Example: find the average of $\frac{1}{2}$ and $\frac{3}{4}$.

$$\begin{aligned} \left(\frac{1}{2} + \frac{3}{4}\right) \div 2 &= \left(\frac{2+3}{4}\right) \times \left(\frac{1}{2}\right) \\ &= \frac{5}{8} \end{aligned}$$

Halfway between $\frac{1}{2}$ and $\frac{3}{4}$, or $\frac{4}{8}$ and $\frac{6}{8}$, is $\frac{5}{8}$.

b) $\frac{13}{4a}$, $a \neq 0$

26. Yes. Example: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and $\frac{1}{2} + \frac{1}{3} = \frac{1}{\frac{6}{5}} = \frac{5}{6}$

$$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} \text{ and } \frac{1}{\frac{x+y}{xy}} = \frac{xy}{x+y}$$

27. a) $\frac{1}{u} + \frac{1}{v} = \frac{u+v}{uv}$ b) 5.93 cm

c) $f = \frac{uv}{u+v}$

28. Step 3 Yes

Step 4a) $A = 2$, $B = 1$

b) $A = 3$, $B = 3$

Step 5 Always:

$$\begin{aligned} \frac{3}{x-4} + \frac{-2}{x-1} &= \frac{3(x-1) - 2(x-4)}{(x-4)(x-1)} \\ &= \frac{x+5}{(x-4)(x-1)} \end{aligned}$$

6.4 Rational Equations, pages 348 to 351

1. a) $4(x-1) - 3(2x-5) = 5 + 2x$

b) $2(2x+3) + 1(x+5) = 7$

c) $4x - 5(x-3) = 2(x+3)(x-3)$

2. a) $f = -1$ b) $y = 6$, $y \neq 0$

c) $w = 12$, $w \neq 3, 6$

3. a) $t = 2$ or $t = 6$, $t \neq 0$ b) $c = 2$, $c \neq \pm 3$

c) $d = -2$ or $d = 3$, $d \neq -4, 1$

d) $x = 3$, $x \neq \pm 1$

4. No. The solution is not a permissible value.

5. a) $\frac{3-x}{x^2} - \frac{2}{x}$, $\frac{3-3x}{x^2}$, $x > 0$

b) $\frac{3-x}{x^2} \times \frac{2}{x}$, $\frac{6-2x}{x^3}$, $x > 0$

c) $x = \frac{1}{2}$

6. a) $b = 3.44$ or $b = 16.56$

b) $c = -3.54$ or $c = 2.54$

7. $l = 15(\sqrt{5} + 1)$, 48.5 cm

8. The numbers are 5 and 20.

9. The numbers are 3 and 4.

10. 30 students

11. The integers are 5 and 6.

12. a) Less than 2 min. There is more water going in at once.

b)

	Time to Fill Tub (min)	Fraction Filled in 1 min	Fraction Filled in x minutes
Cold Tap	2	$\frac{1}{2}$	$\frac{x}{2}$
Hot Tap	3	$\frac{1}{3}$	$\frac{x}{3}$
Both Taps	x	$\frac{1}{x}$	1

c) $\frac{x}{2} + \frac{x}{3} = 1$

d) 1.2 min

13. 6 h

14. a)

	Distance (km)	Rate (km/h)	Time (h)
Downstream	18	$x+3$	$\frac{18}{x+3}$
Upstream	8	$x-3$	$\frac{8}{x-3}$

b) $\frac{18}{x+3} = \frac{8}{x-3}$

c) 7.8 km/h

d) $x \neq \pm 3$

15. 28.8 h

16. 5.7 km/h

17. about 50 km/h west of Swift Current, and 60 km/h east of Swift Current

18. about 3.5 km/h

19.

	Reading Rate in Pages per Day	Number of Pages Read	Number of Days
First Half	x	259	$\frac{259}{x}$
Second Half	$x+12$	259	$\frac{259}{x+12}$

about 20 pages per day for the first half of the book

20. a) 2 L

b) 4.5 L

21. $a = \pm \frac{1}{3}$

22. a) $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$, $x = \frac{2ab}{a+b}$

b) 4 and 12, or -6 and 2

23. a) $\frac{1}{x} - \frac{1}{y} = a$ or $\frac{1}{x} - \frac{1}{y} = a$

$$y - x = axy \quad \frac{y-x}{xy} = a$$

$$y = axy + x \quad y - x = axy$$

$$y = x(ay + 1) \quad y = axy + x$$

$$\frac{y}{ay+1} = x$$

$$y = x(ay+1)$$

$$\frac{y}{ay+1} = x$$

In both, $x \neq 0$, $y \neq 0$, $ay \neq -1$.