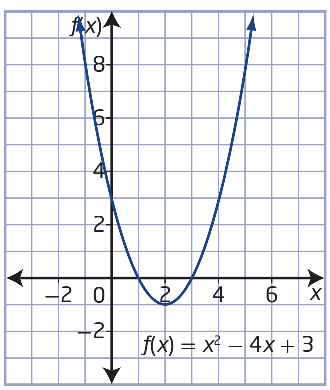
Key Ideas

- The solution to a quadratic inequality in one variable is a set of values.
- To solve a quadratic inequality, you can use one of the following strategies:
  - Graph the corresponding function, and identify the values of x for which the function lies on, above, or below the x-axis, depending on the inequality symbol.
  - Determine the roots of the related equation, and then use a number line and test points to determine the intervals that satisfy the inequality.
  - Determine when each of the factors of the quadratic expression is positive, zero, or negative, and then use the results to determine the sign of the product.
  - Consider all cases for the required product of the factors of the quadratic expression to find any x-values that satisfy both factor conditions in each case.
- For inequalities with the symbol $\geq$ or $\leq$, include the x-intercepts in the solution set.

Check Your Understanding

Practise

1. Consider the graph of the quadratic function $f(x) = x^2 - 4x + 3$.



What is the solution to

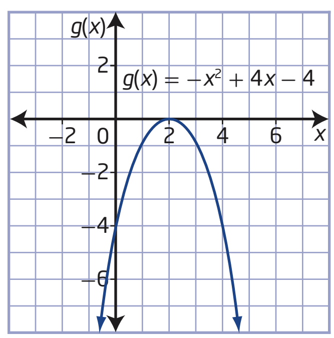
a) $x^2 - 4x + 3 \leq 0$?

b) $x^2 - 4x + 3 \geq 0$?

c) $x^2 - 4x + 3 > 0$?

d) $x^2 - 4x + 3 < 0$?

2. Consider the graph of the quadratic function $g(x) = -x^2 + 4x - 4$.



What is the solution to

a) $-x^2 + 4x - 4 \leq 0$?

b) $-x^2 + 4x - 4 \geq 0$?

c) $-x^2 + 4x - 4 > 0$?

d) $-x^2 + 4x - 4 < 0$?

3. Is the value of x a solution to the given inequality?

a) $x = 4$ for $x^2 - 3x - 10 > 0$

b) $x = 1$ for $x^2 + 3x - 4 \geq 0$

c) $x = -2$ for $x^2 + 4x + 3 < 0$

d) $x = -3$ for $-x^2 - 5x - 4 \leq 0$

4. Use roots and test points to determine the solution to each inequality.

a) $x(x + 6) \geq 40$

b) $-x^2 - 14x - 24 < 0$

c) $6x^2 > 11x + 35$

d) $8x + 5 \leq -2x^2$

5. Use sign analysis to determine the solution to each inequality.

a) $x^2 + 3x \leq 18$

b) $x^2 + 3 \geq -4x$

c) $4x^2 - 27x + 18 < 0$

d) $-6x \geq x^2 - 16$

6. Use case analysis to determine the solution to each inequality.

a) $x^2 - 2x - 15 < 0$

b) $x^2 + 13x > -12$

c) $-x^2 + 2x + 5 \leq 0$

d) $2x^2 \geq 8 - 15x$

7. Use graphing to determine the solution to each inequality.

a) $x^2 + 14x + 48 \leq 0$

b) $x^2 \geq 3x + 28$

c) $-7x^2 + x - 6 \geq 0$

d) $4x(x - 1) > 63$

8. Solve each of the following inequalities. Explain your strategy and why you chose it.

a) $x^2 - 10x + 16 < 0$

b) $12x^2 - 11x - 15 \geq 0$

c) $x^2 - 2x - 12 \leq 0$

d) $x^2 - 6x + 9 > 0$

9. Solve each inequality.

a) $x^2 - 3x + 6 \leq 10x$

b) $2x^2 + 12x - 11 > x^2 + 2x + 13$

c) $x^2 - 5x < 3x^2 - 18x + 20$

d) $-3(x^2 + 4) \leq 3x^2 - 5x - 68$

Apply

10. Each year, Dauphin, Manitoba, hosts the largest ice-fishing contest in Manitoba. Before going on any ice, it is important to know that the ice is thick enough to support the intended load. The solution to the inequality $9h^2 \geq 750$ gives the thickness, h, in centimetres, of ice that will support a vehicle of mass 750 kg.

a) Solve the inequality to determine the minimum thickness of ice that will safely support the vehicle.

b) Write a new inequality, in the form $9h^2 \geq$ mass, that you can use to find the ice thickness that will support a mass of 1500 kg.

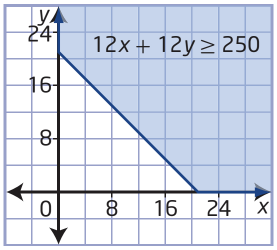
c) Solve the inequality you wrote in part b).

d) Why is the thickness of ice required to support 1500 kg not twice the thickness needed to support 750 kg? Explain.

Did You Know?

Conservation efforts at Dauphin Lake, including habitat enhancement, stocking, and education, have resulted in sustainable fish stocks and better fishing for anglers.

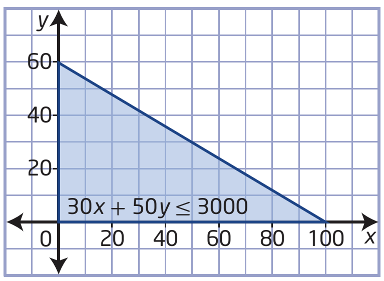
11. a) $12x + 12y \geq 250$, where x represents the number of moccasins sold, $x \geq 0$, and y represents the hours worked, $y \geq 0$.

b) 

c) Example: (4, 20), (8, 16), (12, 12)

d) Example: If she loses her job, then she will still have a source of income.

12. a) $30x + 50y \leq 3000$, $x \geq 0$, $y \geq 0$, where x represents the hours of work and y represents the hours of marketing assistance.

b) 

13. $0.3x + 0.05y \leq 100$, $x \geq 0$, $y \geq 0$, where x represents the number of minutes used and y represents the megabytes of data used; she should stay without a plan if her usage stays in the region described by the inequality.

14. $60x + 45y \leq 50$, $x \geq 0$, $y \geq 0$, where x represents the area of glass and y represents the mass of nanomaterial.

15. $125x + 55y \leq 7000$, $x \geq 0$, $y \geq 0$, where x represents the hours of ice rental and y represents the hours of gym rental.

16. Example:

a) $y = x^2$

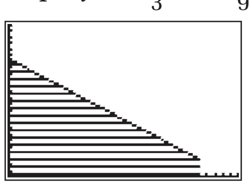
b) $y \geq x^2$; $y < x^2$

c) This does satisfy the definition of a solution region. The boundary is a curve not a line.

17. $y \geq \frac{3}{4}x + 384$, $0 \leq x \leq 512$; $y \leq -\frac{3}{4}x + 384$, $0 \leq x \leq 512$; $y \geq -\frac{3}{4}x + 1152$, $512 \leq x \leq 1024$; $y \leq \frac{3}{4}x - 384$, $512 \leq x \leq 1024$

18. **Step 1** $60x + 90y \leq 35\,000$

Step 2 $y \leq -\frac{2}{3}x + \frac{3500}{9}$, $0 \leq x \leq 500$, $y \geq 0$



(0, 0), approximately (0, 388.9) and (500, 55.6), and (500, 0); y-intercept: the maximum number of megawatt hours of wind power that can be produced; x-intercept: the maximum number of megawatt hours of hydroelectric power that can be produced

Step 3 Example: It would be very time-consuming to attempt to find the revenue for all possible combinations of power generation. You cannot be certain that the spreadsheet gives the maximum revenue.

Step 4 The maximum revenue is \$53 338, with 500 MWh of hydroelectric power and approximately 55.6 MWh of wind power.

19. Example:

	Example 1	Example 2	Example 3	Example 4
Linear Inequality	$y \geq x$	$y \leq x$	$y > x$	$y < x$
Inequality Sign	$\geq$	$\leq$	$>$	$<$
Boundary Solid/Dashed	Solid	Solid	Dashed	Dashed
Shaded Region	Above	Below	Above	Below

20. Example: Any scenario with a solution that has the form $5y + 3x \leq 150$, $x \geq 0$, $y \geq 0$ is correct.

21. a) 48 units^2

b) The y-intercept is the height of the triangle. The larger it gets, the larger the area gets.

c) The slope of the inequality dictates where the x-intercept will be, which is the base of the triangle. Steeper slope gives a closer x-intercept, which gives a smaller area.

d) If you consider the magnitude, then nothing changes.

9.2 Quadratic Inequalities in One Variable, pages 484 to 487

1. a) $\{x \mid 1 \leq x \leq 3, x \in \mathrm{R}\}$

b) $\{x \mid x \leq 1 \text{ or } x \geq 3, x \in \mathrm{R}\}$

c) $\{x \mid x < 1 \text{ or } x > 3, x \in \mathrm{R}\}$

d) $\{x \mid 1 < x < 3, x \in \mathrm{R}\}$

2. a) $\{x \mid x \in \mathrm{R}\}$

b) $\{x \mid x = 2, x \in \mathrm{R}\}$

c) no solution

d) $\{x \mid x \neq 2, x \in \mathrm{R}\}$

3. a) not a solution

b) solution

c) solution

d) not a solution

4. a) $\{x \mid x \leq -10 \text{ or } x \geq 4, x \in \mathrm{R}\}$

b) $\{x \mid x < -12 \text{ or } x > -2, x \in \mathrm{R}\}$

c) $\left\{x \mid x < -\frac{5}{3} \text{ or } x > \frac{7}{2}, x \in \mathrm{R}\right\}$

d) $\left\{x \mid -2 - \frac{\sqrt{6}}{2} \leq x \leq 2 + \frac{\sqrt{6}}{2}, x \in \mathrm{R}\right\}$

5. a) $\{x \mid -6 \le x \le 3, x \in R\}$
b) $\{x \mid x \le -3 \text{ or } x \ge -1, x \in R\}$
c) $\left\{x \mid \frac{3}{4} < x < 6, x \in R\right\}$
d) $\{x \mid -8 \le x \le 2, x \in R\}$

6. a) $\{x \mid -3 < x < 5, x \in R\}$
b) $\{x \mid x < -12 \text{ or } x > -1, x \in R\}$
c) $\{x \mid x \le 1 - \sqrt{6} \text{ or } x \ge 1 + \sqrt{6}, x \in R\}$
d) $\left\{x \mid x \le -8 \text{ or } x \ge \frac{1}{2}, x \in R\right\}$

7. a) $\{x \mid -8 \le x \le -6, x \in R\}$
b) $\{x \mid x \le -4 \text{ or } x \ge 7, x \in R\}$
c) There is no solution.
d) $\{x \mid x < -\frac{7}{2} \text{ or } x > \frac{9}{2}, x \in R\}$

8. a) $\{x \mid 2 < x < 8, x \in R\}$
Example: Use graphing because it is a simple graph to draw.
b) $\left\{x \mid x \le -\frac{3}{4} \text{ or } x \ge \frac{5}{3}, x \in R\right\}$
Example: Use sign analysis because it is easy to factor.
c) $\{x \mid 1 - \sqrt{13} \le x \le 1 + \sqrt{13}, x \in R\}$
Example: Use test points and the zeros.
d) $\{x \mid x \ne 3, x \in R\}$
Example: Use case analysis because it is easy to factor and solve for the inequalities.

9. a) $\left\{x \mid \frac{13 - \sqrt{145}}{2} \le x \le \frac{13 + \sqrt{145}}{2}, x \in R\right\}$
b) $\{x \mid x < -12 \text{ or } x > 2, x \in R\}$
c) $\left\{x \mid x < \frac{5}{2} \text{ or } x > 4, x \in R\right\}$
d) $\left\{x \mid x \le -\frac{8}{3} \text{ or } x \ge \frac{7}{2}, x \in R\right\}$

10. a) Ice equal to or thicker than $\frac{5\sqrt{30}}{3}$ cm, or about 9.13 cm, will support the weight of a vehicle.
b) $9h^2 \ge 1500$
c) Ice equal to or thicker than $\frac{10\sqrt{15}}{3}$ cm, or about 12.91 cm, will support the weight of a vehicle.
d) Example: The relationship between ice strength and thickness is not linear.

11. a) $\pi x^2 \le 630\,000$, where x represents the radius, in metres.
b) $0 \le x \le \sqrt{\frac{630\,000}{\pi}}$ c) $0 \text{ m} \le x \le 447.81 \text{ m}$

12. a) 2 years or more
b) One of the solutions is negative, which does not make sense in this problem. Time cannot be negative.
c) $-t^2 + 14 \le 5$; $t \ge 3$; 3 years or more

13. $\frac{x^2}{2} + x \ge 4$; the shorter leg should be greater than or equal to 2 cm.

14. a) $a > 0$; $b^2 - 4ac \le 0$ b) $a < 0$; $b^2 - 4ac = 0$
c) $a \ne 0$; $b^2 - 4ac > 0$

15. Examples:
a) $x^2 - 5x - 14 \le 0$ b) $x^2 - 11x + 10 > 0$
c) $3x^2 - 23x + 30 \le 0$ d) $20x^2 + 19x + 3 > 0$
e) $x^2 + 6x + 2 \ge 0$ f) $x^2 + 1 > 0$
g) $x^2 + 1 < 0$

16. $\{x \mid x \le -\sqrt{6} \text{ or } -\sqrt{2} \le x \le \sqrt{2} \text{ or } x \ge \sqrt{6}, x \in R\}$

17. a) It is the solution because it is the set of values for which the parabola lies above the line.
b) $-x^2 + 13x - 12 \ge 0$
c) $\{x \mid 1 \le x \le 12, x \in R\}$
d) They are the same solutions. The inequality was just rearranged in part c).

18. They all require this step because you need the related function to work with.

19. Answers may vary.

20. a) The solution is incorrect. He switched the inequality sign when he added 2 to both sides in the first step.
b) $\{x \mid -3 \le x \le -2, x \in R\}$

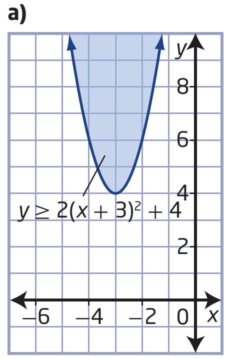
9.3 Quadratic Inequalities in Two Variables, pages 496 to 500

1. a) (2, 6), (−1, 3)
b) (2, −2), (0, −6), (−2, −15)
c) None
d) (−4, 2), (1, 3.5), (3, 2.5)

2. a) (0, 1), (1, 0), (3, 6), (−2, 15)
b) (−2, −3), (0, −8)
c) (2, 9)
d) (−2, 2), (−3, −2)

3. a) $y < -x^2 - 4x + 5$ b) $y \le \frac{1}{2}x^2 - x + 3$
c) $y \ge -\frac{1}{4}x^2 - x + 3$ d) $y > 4x^2 + 5x - 6$

4. a)



b)