#### **Check Your Understanding**

### Practise

**1.** Is each angle,  $\theta$ , in standard position? Explain.



**2.** Without measuring, match each angle with a diagram of the angle in standard position.



**3.** In which quadrant does the terminal arm of each angle in standard position lie?

a)	48°	b)	300°
C)	185°	d)	$75^{\circ}$
e)	220°	f)	160°

**4.** Sketch an angle in standard position with each given measure.

a)	70°	b)	310°
c)	$225^{\circ}$	d)	$165^{\circ}$

**5.** What is the reference angle for each angle in standard position?

a)	$170^{\circ}$	b)	$345^{\circ}$
c)	72°	d)	$215^{\circ}$

**6.** Determine the measure of the three other angles in standard position,  $0^{\circ} < \theta < 360^{\circ}$ , that have a reference angle of

a)	$45^{\circ}$	b)	$60^{\circ}$
c)	30°	d)	$75^{\circ}$

**7.** Copy and complete the table. Determine the measure of each angle in standard position given its reference angle and the quadrant in which the terminal arm lies.

	Reference Angle	Quadrant	Angle in Standard Position
a)	72°	IV	
b)	56°	II	
c)	18°	III	
d)	35°	IV	

**8.** Copy and complete the table without using a calculator. Express each ratio using exact values.

θ	sin θ	cos θ	tan $\theta$
30°			
45°			
60°			

## Apply

**9.** A digital protractor is used in woodworking. State the measure of the angle in standard position when the protractor has a reading of 20.4°.



10. Paul and Gail decide to use a Cartesian plane to design a landscape plan for their yard. Each grid mark represents a distance of 10 m. Their home is centred at the origin. There is a red maple tree at the point (3.5, 2). They will plant a flowering dogwood at a point that is a reflection in the *y*-axis of the position of the red maple. A white pine will be planted so that it is a reflection in the *x*-axis of the position of the red maple. A river birch will be planted so that it is a reflection in both the *x*-axis and the *y*-axis of the position of the red maple.



- a) Determine the coordinates of the trees that Paul and Gail wish to plant.
- **b)** Determine the angles in standard position if the lines drawn from the house to each of the trees are terminal arms. Express your answers to the nearest degree.
- c) What is the actual distance between the red maple and the white pine?

- 11. A windshield wiper has a length of 50 cm. The wiper rotates from its resting position at 30°, in standard position, to 150°. Determine the exact horizontal distance that the tip of the wiper travels in one swipe.
- **12.** Suppose A(x, y) is a point on the terminal arm of  $\angle AOC$  in standard position.
  - a) Determine the coordinates of points A', A", and A''', where



- A' is the image of A reflected in the *x*-axis
- A" is the image of A reflected in the *y*-axis
- A<sup>'''</sup> is the image of A reflected in both the *x*-axis and the *y*-axis
- b) Assume that each angle is in standard position and ∠AOC = θ. What are the measures, in terms of θ, of the angles that have A', A", and A"' on their terminal arms?
- 13. A 10-m boom lifts material onto a roof in need of repair. Determine the exact vertical displacement of the end of the boom when the operator lowers it from 60° to 30°.



14. Engineers use a bevel protractor to measure the angle and the depth of holes, slots, and other internal features. A bevel protractor is set to measure an angle of 72°. What is the measure of the angle in standard position of the lower half of the ruler, used for measuring the depth of an object?



15. Researcher Mohd Abubakr developed a circular periodic table. He claims that his model gives a better idea of the size of the elements. Joshua and Andrea decided to make a spinner for the circular periodic table to help them study the elements for a quiz. They will spin the arm and then name the elements that the spinner lands on. Suppose the spinner lands so that it forms an angle in standard position of 110°. Name one of the elements it may have landed on.



**16.** The Aztec people of pre-Columbian Mexico used the Aztec Calendar. It consisted of a 365-day calendar cycle and a 260-day ritual cycle. In the stone carving of the calendar, the second ring from the centre showed the days of the month, numbered from one to 20.

Suppose the Aztec Calendar was placed on a Cartesian plane, as shown.



- a) The blue angle marks the passing of 12 days. Determine the measure of the angle.
- b) How many days would have passed if the angle had been drawn in quadrant II, using the same reference angle as in part a)?
- c) Keeping the same reference angle, how many days would have passed if the angle had been drawn in quadrant IV?
- **17.** Express each direction as an angle in standard position. Sketch each angle.

<b>a)</b> N20°E	b)	S50°W
-----------------	----	-------

**c)** N80°W **d)** S15°E

# Did You Know?



# **Chapter 2 Trigonometry**

#### 2.1 Angles in Standard Position, pages 83 to 87

- **1.** a) No; the vertex is not at the origin.
  - **b)** Yes; the vertex is at the origin and the initial arm is on the *x*-axis.
  - **c)** No; the initial arm is not on the *x*-axis.
  - **d)** Yes; the vertex is at the origin and the initial arm is on the *x*-axis.



-				
8.	θ	sin θ	cos θ	tan 0
	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
	45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
	60°	$\frac{\sqrt{3}}{2}$	<u>1</u> 2	$\sqrt{3}$

- **9.** 159.6°
- **10. a)** dogwood (-3.5, 2), white pine (3.5, -2), river birch (-3.5, -2)
  - b) red maple 30°, flowering dogwood 150°, river birch 210°, white pine 330°
- **c)** 40 m
- **11.**  $50\sqrt{3}$  cm
- **12. a)** A'(x, -y), A''(-x, y), A'''(-x, -y)**b)**  $\angle A'OC = 360^{\circ} - \theta, \angle A''OC = 180^{\circ} - \theta, \\ \angle A'''OB = 180^{\circ} + \theta$
- **13.**  $(5\sqrt{3} 5)$  m or  $5(\sqrt{3} 1)$  m
- **14.** 252°

18. a)

**15.** Cu (copper), Ag (silver), Au (gold), Uuu (unununium)



Angle	Height (cm)
0°	12.0
15°	23.6
30°	34.5
45°	43.8
60°	51.0
75°	55.5
90°	57.0

- b) A constant increase in the angle does not produce a constant increase in the height. There is no common difference between heights for each pair of angles; for example, 23.6 cm 12 cm = 11.6 cm, 34.5 cm 23.6 cm = 10.9 cm.
- c) When  $\theta$  extends beyond 90°, the heights decrease, with the height for 105° equal to the height for 75° and so on.

**b)** D

- **19.** 45° and 135°
- **20. a)** 19.56 m
- **b) i)** 192° **ii)** 9.13 m

**21. a)** B