9. Solve each equation, for $0° \leq \theta < 360°$, using a diagram involving a special right triangle.

a) $\cos \theta = \frac{1}{2}$ **b)** $\cos \theta = -\frac{1}{\sqrt{2}}$

c) $\tan \theta = -\frac{1}{\sqrt{3}}$ **d)** $\sin \theta = -\frac{\sqrt{3}}{2}$

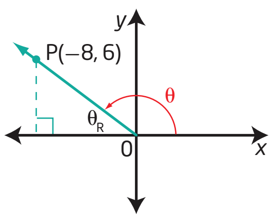
e) $\tan \theta = \sqrt{3}$ **f)** $\tan \theta = -1$

10. Copy and complete the table using the coordinates of a point on the terminal arm.

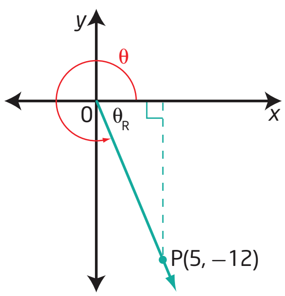
θ	sin θ	cos θ	tan θ
0°			
90°			
180°			
270°			
360°			

11. Determine the values of x, y, r, $\sin \theta$, $\cos \theta$, and $\tan \theta$ in each.

a)



b)



Apply

12. Point P(−9, 4) is on the terminal arm of an angle θ.

a) Sketch the angle in standard position.

b) What is the measure of the reference angle, to the nearest degree?

c) What is the measure of θ, to the nearest degree?

13. Point P(7, −24) is on the terminal arm of an angle, θ.

a) Sketch the angle in standard position.

b) What is the measure of the reference angle, to the nearest degree?

c) What is the measure of θ, to the nearest degree?

14. a) Determine sin θ when the terminal arm of an angle in standard position passes through the point P(2, 4).

b) Extend the terminal arm to include the point Q(4, 8). Determine sin θ for the angle in standard position whose terminal arm passes through point Q.

c) Extend the terminal arm to include the point R(8, 16). Determine sin θ for the angle in standard position whose terminal arm passes through point R.

d) Explain your results from parts a), b), and c). What do you notice? Why does this happen?

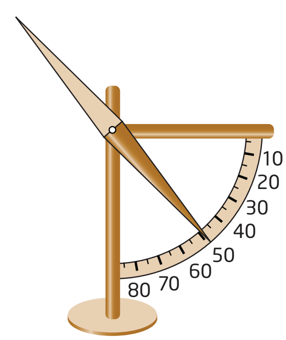
15. The point P(k, 24) is 25 units from the origin. If P lies on the terminal arm of an angle, θ, in standard position, $0° \leq \theta < 360°$, determine

a) the measure(s) of θ

b) the sine, cosine, and tangent ratios for θ

16. If $\cos \theta = \frac{1}{5}$ and $\tan \theta = 2\sqrt{6}$, determine the exact value of sin θ.

17. The angle between the horizontal and Earth's magnetic field is called the angle of dip. Some migratory birds may be capable of detecting changes in the angle of dip, which helps them navigate. The angle of dip at the magnetic equator is 0°, while the angle at the North and South Poles is 90°. Determine the exact values of sin θ, cos θ, and tan θ for the angles of dip at the magnetic equator and the North and South Poles.



18. Without using technology, determine whether each statement is true or false. Justify your answer.

a) $\sin 151° = \sin 29°$

b) $\cos 135° = \sin 225°$

c) $\tan 135° = \tan 225°$

d) $\sin 60° = \cos 330°$

e) $\sin 270° = \cos 180°$

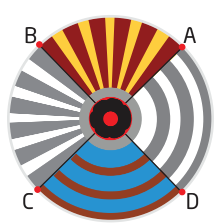
19. Copy and complete the table. Use exact values. Extend the table to include the primary trigonometric ratios for all angles in standard position, $90° \leq \theta \leq 360°$, that have the same reference angle as those listed for quadrant I.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°			
30°			
45°			
60°			
90°			

20. Alberta Aboriginal Tourism designed a circular icon that represents both the Métis and First Nations communities of Alberta. The centre of the icon represents the collection of all peoples' perspectives and points of view relating to Aboriginal history, touching every quadrant and direction.

a) Suppose the icon is placed on a coordinate plane with a reference angle of 45° for points A, B, C, and D. Determine the measure of the angles in standard position for points A, B, C, and D.

b) If the radius of the circle is 1 unit, determine the coordinates of points A, B, C, and D.



21. Explore patterns in the sine, cosine, and tangent ratios.

a) Copy and complete the table started below. List the sine, cosine, and tangent ratios for θ in increments of 15° for $0° \leq \theta \leq 180°$. Where necessary, round values to four decimal places.

Angle	Sine	Cosine	Tangent
0°			
15°			
30°			
45°			
60°			

b) What do you observe about the sine, cosine, and tangent ratios as θ increases?

c) What comparisons can you make between the sine and cosine ratios?

d) Determine the signs of the ratios as you move from quadrant I to quadrant II.

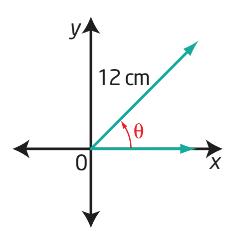
e) Describe what you expect will happen if you expand the table to include quadrant III and quadrant IV.

Extend

22. a) The line $y = 6x$, for $x \geq 0$, creates an acute angle, θ, with the x-axis. Determine the sine, cosine, and tangent ratios for θ.

b) If the terminal arm of an angle, θ, lies on the line $4y + 3x = 0$, for $x \geq 0$, determine the exact value of $\tan\theta + \cos\theta$.

23. Consider an angle in standard position with $r = 12$ cm. Describe how the measures of x, y, $\sin\theta$, $\cos\theta$, and $\tan\theta$ change as θ increases continuously from 0° to 90°.



d) $\cos\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = \frac{\sqrt{2}}{4}$

e) $\sin\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

9. a) 60° and 300° **b)** 135° and 225°
c) 150° and 330° **d)** 240° and 300°
e) 60° and 240° **f)** 135° and 315°

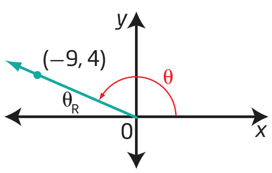
10.

θ	sin θ	cos θ	tan θ
0°	0	1	0
90°	1	0	undefined
180°	0	−1	0
270°	−1	0	undefined
360°	0	1	0

11. a) $x = -8$, $y = 6$, $r = 10$, $\sin\theta = \frac{3}{5}$,
$\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

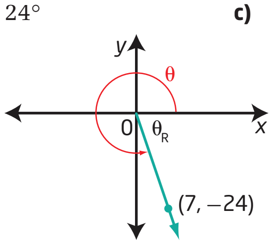
b) $x = 5$, $y = -12$, $r = 13$, $\sin\theta = -\frac{12}{13}$,
$\cos\theta = \frac{5}{13}$, $\tan\theta = -\frac{12}{5}$

12. a)



b) 24° **c)** 156°

13. a)



b) 74° **c)** 286°

14. a) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

b) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

c) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

d) They all have the same sine ratio. This happens because the points P, Q, and R are collinear. They are on the same terminal arm.

15. a) 74° and 106°

b) $\sin\theta = \frac{24}{25}$, $\cos\theta = \pm\frac{7}{25}$, $\tan\theta = \pm\frac{24}{7}$

16. $\sin\theta = \frac{2\sqrt{6}}{5}$

17. sin 0° = 0, cos 0° = 1, tan 0° = 0, sin 90° = 1, cos 90° = 0, tan 90° is undefined

18. a) True. θ_R for 151° is 29° and is in quadrant II. The sine ratio is positive in quadrants I and II.

b) True; both sin 225° and cos 135° have a reference angle of 45° and
$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$.

c) False; tan 135° is in quadrant II, where $\tan\theta < 0$, and tan 225° is in quadrant III, where $\tan\theta > 0$.

d) True; from the reference angles in a 30°-60°-90° triangle,
$\sin 60° = \cos 330° = \frac{\sqrt{3}}{2}$.

e) True; the terminal arms lie on the axes, passing through P(0, −1) and P(−1, 0), respectively, so sin 270° = cos 180° = −1.

19.

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	−1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
180°	0	−1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
225°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	−1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	−1
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
360°	0	1	0

20. a) ∠A = 45°, ∠B = 135°, ∠C = 225°, ∠D = 315°

b) $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$,
$C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

21. a)

Angle	Sine	Cosine	Tangent
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	undefined
105°	0.9659	−0.2588	−3.7321
120°	0.8660	−0.5	−1.7321
135°	0.7071	−0.7071	−1
150°	0.5	−0.8660	−0.5774
165°	0.2588	−0.9659	−0.2679
180°	0	−1	0

b) As θ increases from 0° to 180°, $\sin\theta$ increases from a minimum of 0 to a maximum of 1 at 90° and then decreases to 0 again at 180°. $\sin\theta = \sin(180° - \theta)$. Cos θ decreases from a maximum of 1 at 0° and continues to decrease to a minimum value of −1 at 180°. $\cos\theta = -\cos(180° - \theta)$. Tan θ increases from 0 to being undefined at 90° then back to 0 again at 180°.

c) For $0° \leq \theta \leq 90°$, $\cos\theta = \sin(90° - \theta)$. For $90° \leq \theta \leq 180°$, $\cos\theta = -\sin(\theta - 90°)$.

d) Sine ratios are positive in quadrants I and II, and both the cosine and tangent ratios are positive in quadrant I and negative in quadrant II.

e) In quadrant III, the sine and cosine ratios are negative and the tangent ratios are positive. In quadrant IV, the cosine ratios are positive and the sine and tangent ratios are negative.

22. a) $\sin\theta = \frac{6}{\sqrt{37}}$ or $\frac{6\sqrt{37}}{37}$, $\cos\theta = \frac{1}{\sqrt{37}}$ or $\frac{\sqrt{37}}{37}$, $\tan\theta = 6$

b) $\frac{1}{20}$

23. As θ increases from 0° to 90°, x decreases from 12 to 0, y increases from 0 to 12, $\sin\theta$ increases from 0 to 1, $\cos\theta$ decreases from 1 to 0, and $\tan\theta$ increases from 0 to undefined.

24. $\tan\theta = \frac{\sqrt{1 - a^2}}{a}$

25. Since ∠BOA is 60°, the coordinates of point A are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. The coordinates of point B are (1, 0) and of point C are (−1, 0). Using the Pythagorean theorem $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, $d_{AB} = 1$, $d_{BC} = 2$, and $d_{AC} = \sqrt{3}$.
Then, $AB^2 = 1$, $AC^2 = 3$, and $BC^2 = 4$.
So, $AB^2 + AC^2 = BC^2$.
The measures satisfy the Pythagorean Theorem, so △ABC is a right triangle and ∠CAB = 90°.
Alternatively, ∠CAB is inscribed in a semicircle and must be a right angle. Hence, △CAB is a right triangle and the Pythagorean Theorem must hold true.

26. Reference angles can determine the trigonometric ratio of any angle in quadrant I. Adjust the signs of the trigonometric ratios for quadrants II, III, and IV, considering that the sine ratio is positive in quadrant II and negative in quadrants III and IV, the cosine ratio is positive in the quadrant IV but negative in quadrants II and III, and the tangent ratio is positive in quadrant III but negative in quadrants II and IV.

27. Use the reference triangle to identify the measure of the reference angle, and then adjust for the fact that P is in quadrant III. Since $\tan\theta_R = \frac{9}{5}$, you can find the reference angle to be 61°. Since the angle is in quadrant III, the angle is 180° + 61° or 241°.

28. Sine is the ratio of the opposite side to the hypotenuse. The hypotenuse is the same value, r, in all four quadrants. The opposite side, y, is positive in quadrants I and II and negative in quadrants III and IV. So, there will be exactly two sine ratios with the same positive values in quadrants I and II and two sine ratios with the same negative values in quadrants III and IV.

29. $\theta = 240°$. Both the sine ratio and the cosine ratio are negative, so the terminal arm must be in quadrant III. The value of the reference angle when $\sin\theta_R = \frac{\sqrt{3}}{2}$ is 60°. The angle in quadrant III is 180° + 60° or 240°.

30. **Step 4**

a) As point A moves around the circle, the sine ratio increases from 0 to 1 in quadrant I, decreases from 1 to 0 in quadrant II, decreases from 0 to −1 in quadrant III, and increases from −1 to 0 in quadrant IV. The cosine ratio decreases from 1 to 0 in quadrant I, decreases from 0 to −1 in quadrant II, increases from −1 to 0 in quadrant III, and increases from 0 to 1 in quadrant IV. The tangent ratio increases from 0 to infinity in quadrant I, is undefined for an angle of 90°, increases from negative infinity to 0 in the second quadrant, increases from 0 to positive infinity in the third quadrant, is undefined for an angle of 270°, and increases from negative infinity to 0 in quadrant IV.