

2.1 Review of Factoring

Terminology

Monomial – is a _____ term which can include numbers, variables or both.

Ex: 3 , x^2 , $4x^2y^3$

Binomial – is _____ terms, or two _____ separated with a **sign**.

Ex: $x + 3$, $2x^2 - 1$, $x^2 - 4$, $2x^3y^2 - 5x^4$

Trinomial – is _____ terms, or three _____ separated by **signs**.

Ex: $x^2 - 5x - 6$, $6x^2 - 13x + 6$, $4x^3y + 6x^2 - 3xy$

Polynomial – is _____ of the _____.

A. GCF of a Polynomial:

What to ask yourself to find the GCF:

Number

1. What is the _____ Number that Divides evenly into **ALL** the _____?
Ex: 18 , 24 , 36 $GCF =$ _____

Variable

2. What _____ (s) appears in **ALL OF THE TERMS** and what is the _____ **exponent** of each of these variables?
Ex: x^3y , x^2z , $x^4y^2z^3$ $GCF =$ _____

Example 1: Factor each polynomial.

a) $6x^2 - 15x^3$

b) $24x^2y^3 - 18x^3y$

c) $-3x^3 - 9x^2 + 12x$

B. GCF and Factor by Grouping

If there is no common factor between all of the terms of a polynomial, then we can use the Grouping Method. We can use this method whenever a common factor exists between groups of terms.

Example 2:

a) $3x(x + 5) + 2(x + 5)$

b) $4x(2x - 3) - (2x - 3)$

c) $2x^2 + 8x + 3x + 12$

d) $2x^2 + 3x + 8x + 12$

C. Factoring Simple Trinomials (Polynomials of the form $ax^2 + bx + c$) where $a = 1$

- Find the numbers that multiply to (c) and add to (b)

Example 2: Factor fully each trinomial .

a) $x^2 + 5x + 6$

b) $f^2 - 2f - 8$

c) $-a^2 - 19a + 20$

d) $-4x^2 - 16x + 128$

D. Factoring Complex Trinomials (Polynomials of the form $ax^2 + bx + c$) where $a \neq 1$

To factor trinomials of the form: $ax^2 + bx + c$ when $a \neq 1$ is called: **FACTORING BY DECOMPOSITION (rainbow split)**

- Find the numbers that multiply to $(a \cdot c)$ and add to (b)
- Split the middle term into these two factors
- Factor by grouping

Example 3: Factor each trinomial. Check your answer by expanding.

a) $6x^2 + 17x + 5$

b) $2x^2 - 5x - 3$

c) $12x^2 + 7x + 1$

d) $6a^2 + 20a - 16$

2.1 Worksheet – Review of Factoring

1. Factor. Complete your work on separate paper.

- | | | | |
|-----------------------------|---|-------------------|-------------------------|
| a) $5x + 25$ | b) $8x + 8$ | c) $3x - 15y$ | d) $25x^2 + 10x$ |
| e) $4ax + 8ay - 6az$ | f) $2x^2 - 2x - 6$ | g) $9a^3 + 27b^2$ | h) $12y - 8y^2 + 24y^3$ |
| i) $24xy^2 - 12xy + 36x^2y$ | j) $6m^3n^2 + 18m^2n^3 - 12mn^2 + 24mn^3$ | | |

2. Factor. Complete your work on separate paper.

- | | | |
|---------------------------|---------------------------|---------------------------|
| a) $5x(a + b) + 3(a + b)$ | b) $3m(x - 1) + 5(x - 1)$ | c) $wx + wy + xz + yz$ |
| d) $xy + 4x + 3y + 12$ | e) $x^2 + x - xy - y$ | f) $2x^2 + 6y + 4x + 3xy$ |

3. Factor. Complete your work on separate paper.

- | | | | |
|--------------------|----------------------|---------------------|-------------------------|
| a) $x^2 + 5x + 4$ | b) $x^2 + 8x + 15$ | c) $m^2 - 10m + 16$ | d) $y^2 - 13y + 42$ |
| e) $x^2 - x - 20$ | f) $n^2 - 10n - 24$ | g) $x^2 - 4x - 21$ | h) $x^2 + 12xy + 35y^2$ |
| i) $2x^2 - 6x + 4$ | j) $5y^2 + 40y + 60$ | k) $4m^2 - 8m - 60$ | l) $x^3 + 18x^2 + 72x$ |

4. Find 3 values of k such that the trinomial can be factored over the integers.

$$x^2 - 5x + k$$

5. List all values of k for which the trinomial can be factored over all integers.

$$x^2 + kx - 12$$

6. Factor fully,

- | | | | |
|----------------------|-----------------------|----------------------|-----------------------|
| a) $2x^2 + 5x + 2$ | b) $3x^2 - 5x - 2$ | c) $6m^2 + 13m - 5$ | d) $3x^2 + x - 4$ |
| e) $2y^2 + 11y + 15$ | f) $12m^2 - 11m + 2$ | g) $4x^2 - 18x - 10$ | h) $12n^2 + 27n + 15$ |
| i) $6y^2 + 14y - 12$ | j) $6x^2 + xy - 2y^2$ | | |

2.2 Factoring Special Polynomials

A. Factoring PERFECT SQUARE TRINOMIALS

A perfect square trinomial has the form: _____

Perfect square trinomials have the following characteristics:

- a and c must be perfect squares (1, 4, 9, 16, etc....)
- bx is equal to twice the product of the square roots of terms a and c .

Example 1: Verify that the following trinomials are perfect square trinomials.

a) $4x^2 + 12x + 9$

b) $9x^2 - 6x + 1$

Example 2: Factor the following trinomials

a) $4x^2 + 4x + 1$

b) $4x^2 - 12x + 9$

c) $4 - 20x + 25x^2$

d) $x^2 - 2x + 1$

e) $100x^2 - 140xy + 49y^2$

B. Factoring the DIFFERENCE OF SQUARES

The difference of squares has the form: _____

The difference of squares has the following characteristics:

- There are only two terms in the polynomial (binomial)
- Each term is a perfect square
- Second term must be subtracted from the first term

Example 3: Factor the following binomials

a) $16x^2 - 25$

b) $49n^2 - 121$

c) $x^4 - 1$

d) $5x^4 - 80y^4$

e) $(x - 3)^2 - 16$

f) $w^2 - 5$

2.2 WS Factoring Special Polynomials

Complete on a separate sheet of paper. Show work where possible. Perfect Square trinomials must be checked if the shortcut is used.

1. Identify polynomial type as i) simple trinomial, ii) complex trinomial, iii) difference of squares, iv) perfect square trinomial or v) none of the above

a) $y^2 - 10y + 25$ b) $49 - f^2$ c) $4t^2 + 4t + 1$ d) $36y^2 + 49$

e) $16x^2 + 24x - 9$ f) $100p^2 - 121q^2$ g) $49y^2 - 126y + 81$ h) $2x^2 - 32$

2. Factor, if possible.

a) $x^2 - 9$ b) $y^2 - 64$ c) $x^2 + 81$ d) $x^2 + 6x + 9$

e) $y^2 - 10y + 25$ f) $49 - f^2$ g) $4t^2 + 4t + 1$ h) $36y^2 - 49$

i) $16x^2 + 24x + 9$ j) $100p^2 - 121q^2$ k) $49y^2 - 126y + 81$ l) $2x^2 - 32$

m) $3y^2 + 6y + 3$ n) $36x^2 - 81y^2$ o) $y^3 - 18y^2 + 81y$ p) $3x^2 - 48x$

q) $3x^3 - 48x$ r) $\frac{x^2}{64} - \frac{y^2}{121}$ s) $5m^3 - 40m^2 + 80m$ t) $\frac{2x^2}{9} - 2$

u) $2x^4 - 32$

2.3 Factoring Composite Quadratic Trinomials and Higher Ordered Polynomials

A. Composite Quadratic Trinomials

A composite quadratic trinomial has the form: _____

A composite quadratic trinomial has the following characteristics:

- There are three terms in the polynomial
- The first and second terms have the same binomial
- The first term's binomial is squared

Steps for factoring a composite quadratic trinomial

- 1) Replace the binomial with a different variable
- 2) Factor the resulting trinomial
 - If $a = 1$, factor the simple trinomial
 - If a does not = 1, factor by decomposition

Example 1: Factor

a) $(x - 3)^2 + 6(x - 3) + 8$

b) $(x + 1)^2 + (x + 1) - 72$

c) $2(x - 1)^2 - 5(x - 1) - 3$

d) $12(x + 2)^2 + 5(x + 2) - 2$

B. Higher Ordered Polynomials

A higher ordered polynomial, in Pre-Calculus 11, will be limited to the following:

- The highest order will be even
- Mostly trinomials

Example: Factor

a) $x^4 + 4x^2 + 3$

b) $x^6 - x^3 - 12$

c) $6x^4 - 11x^2 - 35$

d) $x^4 - 81$

2.3 WS Composite and Higher Order Polynomials

Complete your work on a separate sheet of paper. Show your work.

1. Factor fully.

a) $(x + 1)^2 + 3(x + 1) + 2$

b) $(x - 1)^2 + 3(x - 1) + 2$

c) $(x + 3)^2 + 4(x + 3) - 5$

d) $(x + 2)^2 - 20(x + 2) + 36$

e) $(x - 5)^2 - 4(x - 5) - 77$

f) $2(x + 1)^2 - 6(x + 1) + 4$

g) $4(x + 1)^2 + 8(x + 1) + 3$

h) $10(x + 1)^2 - 3(x + 1) - 1$

i) $2(x + 1)^2 + 7(x + 1) - 30$

j) $(x - 5)^2 - 16$

2. Factor fully.

a) $x^4 + 2x^2 + 1$

b) $x^4 + x^2 - 6$

c) $x^6 - 3x^3 - 10$

d) $x^4 + x^2y^2 - 12y^4$

e) $x^4 - 11x^2y + 24y^2$

f) $x^{10} + 2x^5 - 63$

Unit 2: Factoring Polynomials – Review

Calculators are permitted. Show work when needed. If factoring Perfect square Trinomials using the "shortcut", check the middle term.

1. Factor each polynomial.

a) $4m^2 - 28m$

b) $3ab - 9ab^2 + 6a^2b$

c) $-21y^2x - 7y^2$

d) $80m^6 + 10m^2n^2 + 80m^2$

e) $-24a^5 + 40a^2 + 8a + 56$

f) $-21x^4 - 9x^3 + 9x^2$

2. Factor each polynomial.

a) $y^2 + 2y + 2x + xy$

b) $x^2 + 5x + 2x + 10$

c) $3x(x - 2) - (x - 2)$

d) $m(m + 3) - 4(m + 3)$

e) $d^2 - 4d + 3d - 12$

f) $r(2r - 5) + 3(2r - 5)$

3. Factor each trinomial.

a) $x^2 + x - 20$

b) $x^2 - 7x + 12$

c) $c^2 + 4c - 21$

d) $x^2 - 6x - 27$

e) $2x^2 + 12x + 18$

f) $3m^2 - 3m - 6$

4. Factor each trinomial.

a) $3y^2 + 4y - 7$

b) $8x^2 - 6x - 5$

c) $3c^2 + 8c + 5$

d) $6x^2 - x - 2$

e) $8x^2 + 8x - 6$

f) $2m^2 - 9m + 4$

5. The face of a Canadian \$20 bill has an area that can be represented by the expression $10x^2 + 9x - 40$.

a) Factor $10x^2 + 9x - 40$ to find expressions that represent the dimensions of the bill.

b) If x represents 32 mm, what are the dimensions of the bill, in millimetres?

6. Find an integer to replace " k " so that each trinomial is a perfect square trinomial.

a) $36x^2 + kx + 25$

b) $9x^2 + 30x + k$

7. Factor each trinomial.

a) $4r^2 - 4r + 1$

b) $x^2 + 8x + 16$

c) $4m^2 + 20m + 25$

8. Factor fully.

a) $x^2 - 121$

b) $1 - 49m^2$

c) $27m^3 - 12m$

d) $3x^2 - 27$

e) $8x^2 - 18y^2$

c) $\frac{1}{4}x^2 - \frac{9}{25}y^2$

9. Factor each expression.

a) $(x + 2)^2 - (x + 2) - 42$

b) $(x + 5)^2 + 8(x + 5) + 12$

c) $4(y + 1)^2 - 7(y + 1) + 3$

d) $4(5v - 3)^2 + 10(5v - 3) - 6$

e) $4 - (x - 3)^2$

f) $(2x - 5)^2 - 144$

10. Factor.

a) $4x^4 - 81$

b) $x^4 + x^2 - 20$

c) $4x^6 - 12x^3 + 9$

d) $8x^4 + 10x^2 + 3$

e) $2x^{10} - 11x^5 + 15$

f) $5x^4 - 80$