

5.1/5.2 – Solving Systems of Equations (by Graphing)

Problem: My dad is 2 years older than my mom. The sum of their ages is 132 years. How old are they?

A linear system of equations is _____

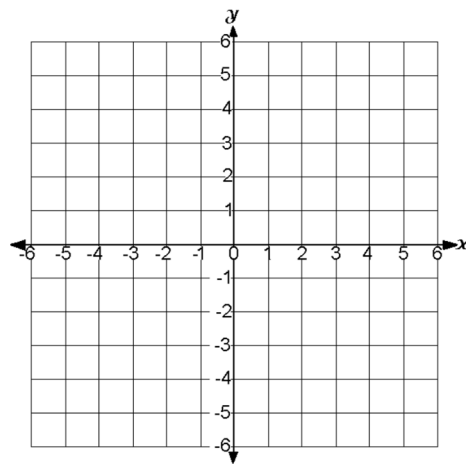
The solution to a linear system is _____

Example #1: Determine the solution to the linear system.

$$y = -\frac{2}{3}x + 1$$

$$y = x - 4$$

Proof/check:

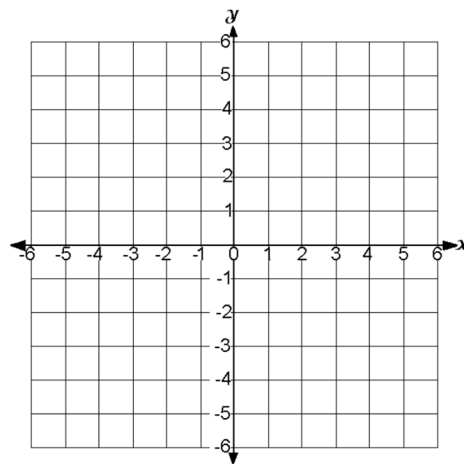


Example #2 : Determine the solution to the linear system.

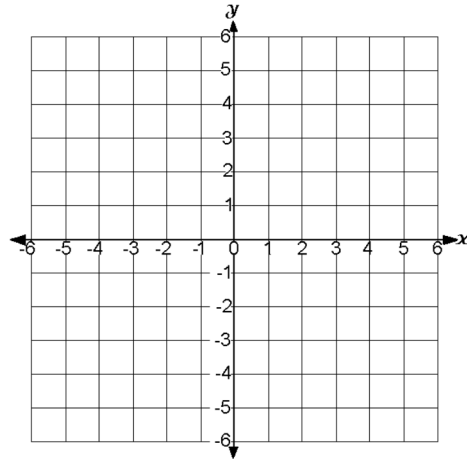
$$3x + 2y = -12$$

$$-2x + y = 1$$

Method #1: $y = mx + b$



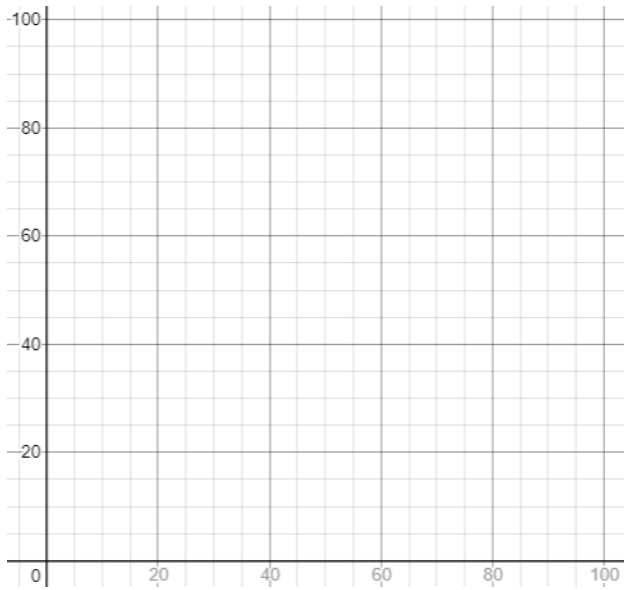
Method #2: *x* and *y* Intercepts



Example #3: Bill received and sent 60 text messages in one weekend. He sent 10 more messages than he received.

a) Write a linear system to model this situation.

b) Graph the linear system then solve the problem. How many text messages did Bill send and how many did he receive?



5.3 - Solving Systems of Equations by Substitution

We can eliminate a variable by solving one equation for a variable and substituting it into the other one.

Substitution Steps:

1. Solve for one variable from either equation (it is simplest to select the variable whose coefficient is one).
2. Substitute the result from step 1 into the other equation.
3. Solve for the remaining variable.
4. Substitute known value into either original equation and solve.
5. Check your answer.

Example 1:

$$3x + y = 3$$

$$7x - 2y = 20$$

Example 2:

$$9x + 6y = -7$$

$$3x - 3y + 4 = 0$$

Example 3:

$$\frac{3}{4}x + \frac{1}{2}y = -\frac{7}{12}$$

$$\frac{4}{3} + x = y$$

Recap Steps:

Substitution Assignment

Is (2, -3) a solution to the system? Show work for verification or check of each equation.

$$\begin{aligned} 1) \quad & -4x + y = -6 \\ & -x + 4y = 6 \end{aligned}$$

$$\begin{aligned} 2) \quad & 6x + 8y = -12 \\ & x - 6y = 20 \end{aligned}$$

Solve the following systems by substitution method.

$$\begin{aligned} 3) \quad & y = -2x - 1 \\ & 5x + 2y = -6 \end{aligned}$$

$$\begin{aligned} 4) \quad & -x + 2y = -13 \\ & -3x + y = -4 \end{aligned}$$

Continue assignment on back.

$$\begin{aligned} 5) \quad & -7x + 5y = -14 \\ & 7x - 2y = -7 \end{aligned}$$

$$\begin{aligned} 6) \quad & 2x + 4y = -12 \\ & 6x + 4y = -4 \end{aligned}$$

5.4 – Solving Systems of Equations by Elimination

When we solve a linear system by graphing, it is not always possible or convenient to get exact values. We will look at another algebraic method that can be used to find these exact values.

Properties of Linear Systems

1. Multiplying both sides of either equation of a linear system by a constant does not change the solution.
2. Adding or subtracting the equations of a linear system does not change the solution.

Elimination Steps:

1. One of the variables must have the same coefficient (number in front) in both equations; if not, multiply one or both of the equations to create equal coefficients.
2. Add or subtract the equations to eliminate one of the variables.
3. Solve for the remaining variable.
4. Substitute known value into either original equation.
5. Solve for remaining variable.
6. Check your answer.

Example 1:

$$15x - 2y = 9$$

$$5x + 4y = 17$$

Example 2:

$$2x - 5y = 29$$

$$3y = 7x$$

Example 3:

$$\frac{2}{3}x - \frac{1}{2}y = 4$$

$$\frac{1}{2}x - \frac{5}{2} = -\frac{1}{4}y$$

Recall Steps:

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Elimination Assignment

Name _____

Block _____

Check if (3, 1) is a solution to the system? Show all your work for verifying each equation.

1) $x - 2y = 5$
 $-x - 4y = 1$

2) $-x + 3y = 0$
 $6x - 3y = 15$

Solve each system using elimination method.

3) $-3x - 5y = -13$
 $-3x - 6y = -15$

4) $4x - 9y = 8$
 $-x - 3y = -2$

$$5) -2x - 6 = 5y$$

$$\frac{5}{6}x = 1 - \frac{1}{3}y$$

$$6) -54 + 9x - 18y = 0$$

$$8y - 4x = -16$$

Name : _____ Block: _____

Unit 5 Checkpoint #1 (5.1 – 5.4)

1. Identify the variables and create a linear system to model the situation.
 - a) The total mass of vitamin C in one apple and two peaches is 17 mg. The total mass of vitamin C in two apples and one peach is 13 mg.

 - b) Sea otters live along the coasts of California and Northern BC. The total number of sea otters is 130 000. There are twenty-five times more sea otters in Northern BC than in California.

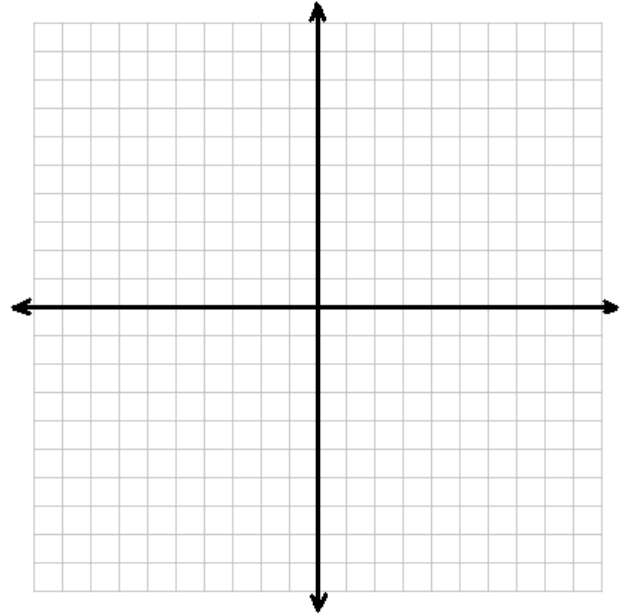
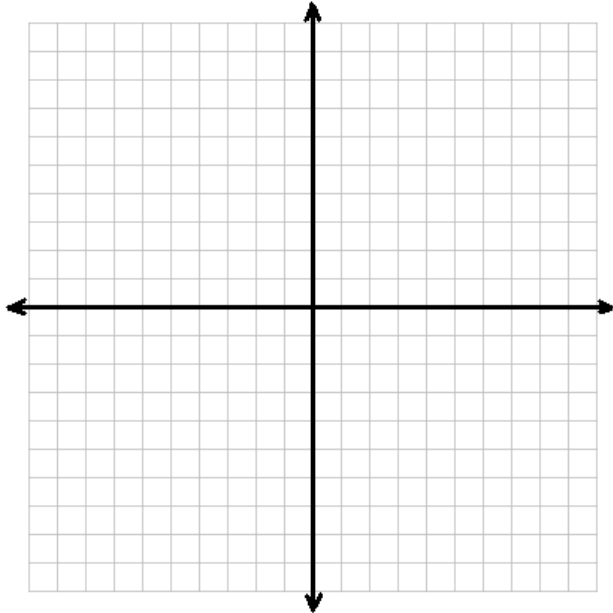
 - c) The cost for theatre tickets is \$20 for adults and \$12 for students. \$9184 was collected at the box office. 550 people attended the show.

2. Verify that $(2, -1)$ is a solution to the linear system.
$$2x + y = 3$$
$$4x + 3y = 5$$

3. Solve the linear systems by graphing.

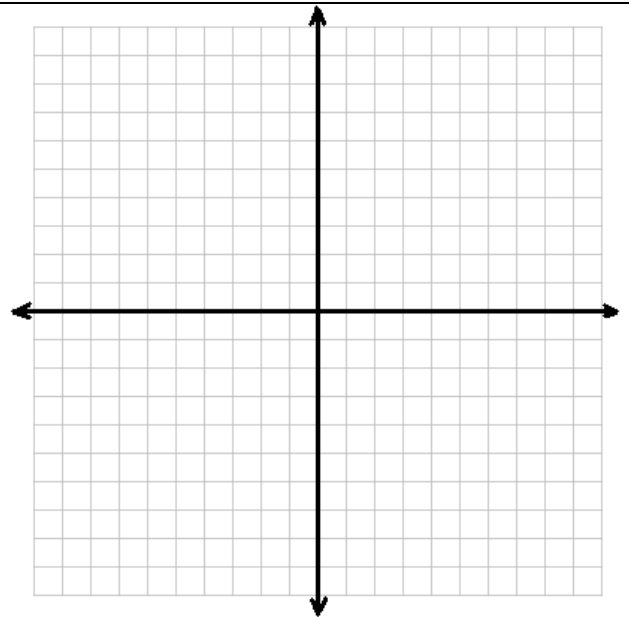
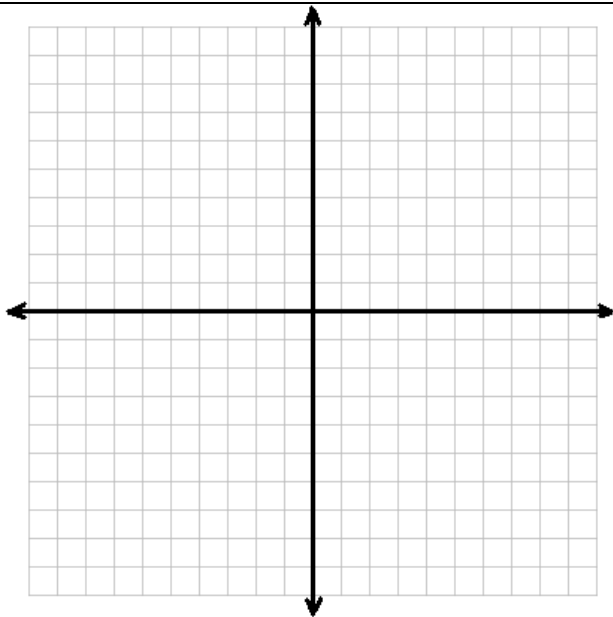
a) $2x - y = 1$
 $x + y = 5$

b) $x - 2y + 2 = 0$
 $x - y + 1 = 0$



c) $2x - 3y = 0$
 $4x + 3y = 18$

d) $y = \frac{1}{3}x + 1$
 $x - y = 3$



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4. Solve the system of equations using the substitution method.

a) $x + 4y = 6$
 $2x - 3y = 1$

b) $x - y = 1$
 $3x + y = 11$

c) $3x + 4y = 15$
 $x + y = 5$

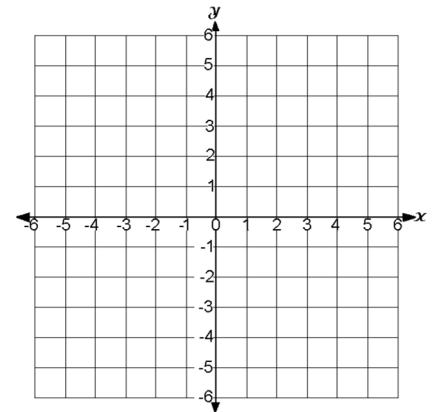
5.5 – Properties of Linear Systems

The Number of Solutions of a Linear System

Ex#1: Graph the following system of equations:

$$-3x + y = 5$$

$$-6x + 2y = 10$$



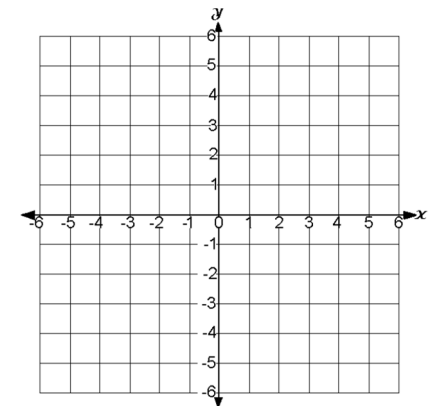
What do you notice about the graphs? (slopes? intercepts?)

Where does the first equation intersect with the second equation? What is the solution? How many solutions are there to this system?

Ex#2: Graph the following system of equations:

$$2x - 5y = 10$$

$$2x - 5y = -5$$



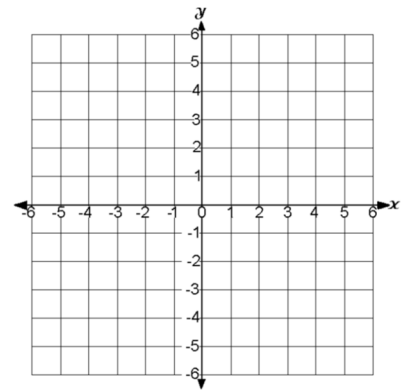
What do you notice about the graphs? (slopes? intercepts?)

How many solutions to this system? How are the equations alike? How are they different?

Ex#3: Graph the following system of equations:

$$2x - 3y = 3$$

$$-2x + y = 3$$



What do you notice about the graphs?

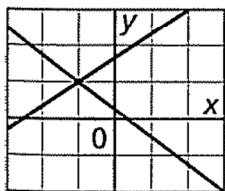
At how many points does the graph of the first equation meet the graph of the second? How are the equations alike? How are they different?

Summary:

Possible Solutions for a Linear System

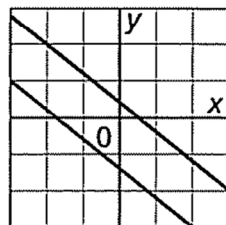
Intersecting Lines

One solution



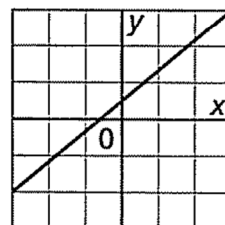
Parallel Lines

No solution



Coincident Lines

Infinite solutions



Solve all questions on a **separate** piece of paper. Show your work clearly.

1. Solve by **graphing**. Show the graphs on graph paper.

a) $3x + y = -1$

$y = x + 3$

b) $2x - 4y = 8$

$y = 2x + 1$

2. Solve by **substitution**.

a) $4x + y = -6$

$-2x + 3y = 24$

b) $2x + y = 9$

$x - y = 3$

c) $-3x - 4y = -2$

$x + 2y = 3$

3. Solve by **elimination**.

a) $2x + 3y = 6$

$5x + 10y = 20$

b) $3a + 10b = -4$

$4a - 5b = 13$

c) $2x - 9 = -5y$

$-2y + 3x = 4$

d) $x - \frac{1}{3}y = \frac{4}{3}$

$\frac{5}{6}x + \frac{1}{2}y = \frac{3}{2}$

4. Solve the following **word problems**. Be sure to write a Let $x =$ statement. Write a system of linear equations to model the situation. Solve by method of your choice.

a) At a sale, all DVD's are on sale at one price and all Xbox games at another price. Three DVD's and 2 Xbox games cost \$72. One DVD and 3 Xbox games cost \$52. Find the cost of a DVD and an Xbox game.

b) A video game club charges an annual fee and an hourly fee. In one year, Jill played a total of 15 hours and paid \$235. That same year, Karl played for 18 hours and paid \$262. Find the annual fee and the hourly fee.

c) The sum of two numbers is 64. Their difference is 14. Find the two numbers.

d) The perimeter of a rectangle is 384m. The length is 82m longer than the width. Find the length and the width.

e) A playoff football game drew 36 500 fans. Depending on seat location, some seats sold for \$35 and some for \$20. The total income from ticket sales was \$940 000. Determine the number of \$35 tickets and \$20 tickets sold?

f) The final exam consists of multiple choice and written questions. The total number of questions on the exam is 85. Jane scored 80% on the multiple choice section and 70% on the written section. Jane scored a total of 61 point on the test. Determine the number of multiple choice and written questions on the final exam?

5. Without graphing, determine the number of solutions to the equations below. One, infinite or none. Justify your choice.

a) $y = \frac{5}{3}x + 2$

$5x - 3y - 12 = 0$

b) $5x - 3y = 12$

$10x - 6y - 24 = 0$

c) $2x + y = 5$

$4x + y = 9$

Solutions:

1a) (-1,2) b) (-2,-3)

2a) (-3,6) b) (4,1) c) $(-4, \frac{7}{2})$

3a) (0,2) b) (2,-1) c) (2,1) d) $(\frac{3}{2}, \frac{1}{2})$

4a) DVD = \$16 Xbox = \$12 b) Annual = \$100 Hourly = \$9 c) Larger # = 39 Smaller # = 25

d) Length = 137m Width = 55m e) 14 000 tickets @ \$35 22 500 tickets @ \$20

f) Multiple Choice = 15 Written = 70

5a) none b) infinite c) one