8.1 – Simple Interest

Borrowing money is *not* free. Usually what you pay to the lender is a percentage of the money you borrowed. This is called _____ .

If you deposit money into the bank, you are essentially lending the bank money and in exchange they pay you interest. The bank often pays you much _____ interest for borrowing your money than they would charge you for borrowing there's.
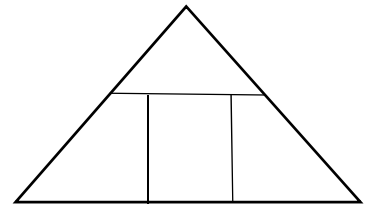
There are basically two ways interest is calculated:

1. Interest on the principal amount only is called _____ interest and the formula to calculate this is…

Simple Interest Formula

$$I = Prt$$

I = Interest
P = Principal (Initial Value)
r = Interest Rate
t = time (years)

$$A = P + I$$

2. When the interest accumulated from each interest period is added to the principal and interest calculated on that amount, it is called _____ interest. We will look more into compound interest later.

Note: the interest rate is expressed as a decimal and the time is expressed in years.

Think about the values of *r* at:

a) 3%   b) 0.9%   c) $\frac{1}{6}$%

Think about the values of *t* at:

a) 2 years   b) 3 months   c) 10 days

Example 1: If Sam were to invest $350.00 at 7% per year for 16 months. Calculate:

   a) the simple interest   b) the amount of the investment at the end of 16 months

Example 2: If Aya acquired a $2000.00 loan for 8 months and was required to pay $64 in interest. What was the annual rate of interest?

Example 3: If Don made a 3 year investment at 4.5% interest and earned $675. How much was the original investment?

Example 4: What principal must be invested now at 1.25% annual simple interest to have an amount of $174 in 2 years?

8.1 WS – Simple Interest

1. A principle of $500 was invested for 3 months at 5.5% annual simple interest. Calculate the simple interest earned on this investment.

2. A principal was invested for 5 years at 5.27% annual interest and earned $51.25 of interest. Calculate the principal. Round the principal to the nearest cent.

3. An investment of $450 accumulates $21.43 of simple interest in the account after 15 months. What is the annual interest rate on the investment? Give the answer to 2 decimal places.

4. Abbie received $1000 from a $5000-bond that earned simple interest at a rate of 4% annually. For how long was the money invested?

5. A principal of $1575 was invested and the interest earned is $171.05. What is the amount of the investment?

6. The amount of an investment is $1206.58 and the principal is $1150. What is the interest earned?

7. A principal of $700 was invested for 5 years at 5% annual simple interest. What is the amount of the investment after 5 years?

8. What principal must be invested now at 4.3% annual simple interest to have an amount of $265 in 5 years?

9. A credit card company charges an interest rate of 22% per year on outstanding balances. What interest was charged on an overdue bill of $618.08 for 50 days?

10. At a 4% annual simple interest rate, how long would it take an investment of $4200 to grow to 4485.6?

11. Brad invested $1500 in a savings account earning simple interest.
    At the end of 3.5 years, he had a total of $1815 in his account.
    a) How much simple interest did he earn in 3.5 years?
    b) How much simple interest did he earn per year?
    c) What was the rate of simple interest per year for his account? Explain your work.

8.2 – Compound Interest

Two kinds of Canada Savings Bonds (CSB) are regular and compound bonds. Regular Canada Savings Bonds earn simple interest that is deposited into the owner's bank account each year. Compound Canada Savings Bonds earn compound interest and the total amount of the bond is paid when it is cashed.

Consider the growth of a $500 CSB of each type at and interest rate of 5% over a 5-year period:

Regular CSB					Compound CSB			
Year	P($)	I($)	A($)		Year	P($)	I($)	A($)
1	500	25	525		1	500	25	525
2	500				2	525		
3	500				3			
4	500				4			
5	500				5			

Which CSB type would you choose and why?

_____

_____

When interest is earned or paid on interest, the interest compounds. This is known as compounding interest and the formula used to calculate it is:

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

Where,

P = principal amount

r = annual nominal interest rate (as a decimal)

n = number of times the interest is compounded per year

t = number of years

Types of compounding	Number of compounding periods
Annually	
Semi-annually	
Monthly	
Bi-weekly	
Weekly	
Daily	

Example 1: $7000 is invested in a 6 year GIC compounded quarterly at a rate of 5% per annum. Determine the value of the investment at the end of the term.

Example 2: RBC and TD offer the following investment opportunities for an initial investment of $10000:

- RBC pays interest at an annual rate of 7.3% compounded annually.
- TD pays interest at an annual rate of 7.2% compounded monthly.

a) Which bank provides the greater interest at the end of:
    i.   1 year   ii.   30 years

Example 3: Roz received a loan for $2500 for 4 years compounded bi-monthly and paid $842.26 in interest. What was the annual rate of interest to the nearest tenth of a percent?

Example 4: Andy wants to invest some money with the goal of having $8000 in 5 years. The bank offers an annual rate of 5.7% compounded weekly. How much should Andy's initial investment be?

Example 5: A GIC pays 3.49% interest, compounded annually. A principal of $5000 is invested. Approximately how long (to the nearest whole number) will it take for the investment to:
    a) double?   b) reach a value of $20000?

Rule of 72:

$$\frac{72}{\text{Rate of Return}} = \text{Time for Investment to Double}$$

8.2 WS – Compound Interest

1. A principal of $2000 is invested that earns interest at 5.5% compounded annually. Determine the amount after 2 years.

2. A principal of $2700 is deposited in an account that earns 6.1% interest, compounded weekly. What is the amount after 4 years?

3. There is a principal of $3900 in a savings account earning interest compounded annually. What must the annual interest rate be for this principal to double in 25 years?

4. Jamie would like to have $5000 saved after 3 years. The savings account he chooses pays 5.9% interest, compounded semi-annually. What principal does Jamie need to invest today?

5. A principal of $4500 is invested for 9 years at an interest rate of 4.9% compounded quarterly. Calculate the interest earned.

6. A principal of $3650 is invested for 7 years and earns interest of $1284. The interest rate is compounded quarterly. Calculate the annual interest rate, to 2 decimal places.

7. An investment account pays 9% interest, compounded monthly. How much money would you need to invest in this account at age 22, to retire with $400 000 at age 60?

8. Currently, there is a principal of $2000 in a savings account. What is the approximate annual interest rate for this principal to double in 15 years?

9. Carrie borrowed $3350 at 5.7% interest, compounded quarterly. The loan requires no payment until the end of the 3-year term.
   a) What amount does Carrie pay after 3 years?   b)   How much interest does Carrie pay?

10. Which of these two investments give the higher return at the end of one year given an initial investment of $1000? Justify your answer.

   - Option A: 5.48% compounded monthly
   - Option B: 4.28% compounded quarterly

<u>Checkpoint</u>: ***Can you . . .***

- o calculate the simple interest and principal for an investment or loan?
- o calculate the interest rate and time for an investment or loan?
- o calculate the simple interest and amount for an investment or loan?
- o calculate the principal, given the amount of an investment earning simple interest?
- o calculate the amount for interest compounded annually?
- o calculate the amount and principal for any compounding period?
- o calculate the interest for any compounding period?
- o use the rule of 72 to estimate the time for an investment to double?

1. Mike invested $2000 that earned $45.21 in simple interest over 30 months.
   a) What is the value of Mikes investment after 30 months?

   b) What was the annual rate of interest?

2. Mariko invested $800 at 5% per annum and received $20 simple interest. How long was her money invested for?

3. A loan was repaid after 6 months. The simple interest paid on the loan was $19.50. The annual interest rate was 7.8%. What was the initial amount of the loan?

4. You are considering a short-term loan to purchase textbooks for university of $1500 at 8% p.a. Working through the summer, you will be able to pay the loan off in 9 months. What will you owe after 9 months and how much interest did you have to pay?

5. The amount to be repaid on a loan is $5500. The interest on the loan is $500 and the simple interest rate is 3.75% annually.
   a) How much money was borrowed?

   b) For what length of time was the money borrowed?

6. $8000 is invested in an RRSP for 7 years compounded quarterly at a rate of 9.2% per year. Determine the value of the investment at the end of the term.

7. Mr. Mathers wanted to invest some money so that his daughter will have $20000 for her college education in 10 years time. The bank offered him an annual rate of 6.8% compounded semi-annually. How much should his initial investment be?

8. Lennon borrowed $3400 for 3 years and paid interest compounded bi-weekly. No payments were required until the end of the 3 years. If at the end of the term he repaid $4670.39, determine the annual rate of interest (to the nearest tenth of a percent).

9. Two banks offer GICs:

   o Bank A offers a return of 5.3%, compounded annually for 5 years.
   o Bank B offers a return of 6% simple interest for 5 years.

   a) Which GIC earns more interest?   b) How much more interest?

8.3 – Annuities: Investments and Loans

Warm up: Solve for x in the equation: $y = a(x - p)^2 + q$

An **annuity** is a series of equal deposits (or payments), equally distributed over time. The 2 common types of annuities that we'll look at are:

Investment annuity: where there is a certain principal deposited and then regular payments made over the course of the investment

Loan: where you make loan payments on a regular schedule (every month, year, quarter, etc.) and are paying interest on the loan

The amount of an annuity is also called the *future value* of the annuity.

$$A = \frac{R\left[(1 + i)^n - 1\right]}{i}$$

- A is the amount in dollars.
- R is the regular deposit or payment in dollars.
- i is the interest rate per compounding period, as a decimal.
- n is the number of deposits or payments.

$$R = \frac{Ai}{(1 + i)^n - 1}$$

Example 1: A regular deposit of $100 is invested in a savings account at the end of each month. The interest rate is 2% compounded monthly. What is the amount of the annuity at the end of 50 years?

Example 2: Suppose you want to retire at age 60 with $1 000 000. At age 20, you start investing a monthly deposit in a stock portfolio that pays 4.6% interest compounded monthly. What should your monthly deposit be to achieve your goal?

An annuity can also be used to make *regular payments* in the future. For example, a sum of money can be invested now to provide regular equal payments in the future that can be used to provide a source of income after you retire. The money that is invested <u>now</u> is called the _____ of the annuity.

Present Value of an Annuity

The present value of an annuity is: $PV = \dfrac{R\left[1 - (1 + i)^{-n}\right]}{i}$

- PV is the present value in dollars.
- R is the regular payment in dollars.
- i is the interest rate per compounding period, as a decimal.
- n is the number of payments.

$R = \dfrac{PVi}{1 - (1 + i)^{-n}}$

The **present value** of an annuity is the sum that must be invested now to guarantee a desired payment in the future, while the **amount** (**future value**) of an annuity is the total which will be achieved over time.

Example 3: A person buys a sound system. She pays $65 a month for 48 months at an interest rate of 22% compounded monthly. What is the present value of the loan?

Example 4: A person has a $50 000 student loan. The loan is repaid over 7 years. The bank charges 6.5% interest compounded quarterly.

   a) What is the quarterly repayment on the loan?

   b) How much interest did they pay?

HW: 8.3 WS

8.3 WS - Annuities

1. Determine the amount of each annuity.
a) annual payments of $545, for 17 years, at 5.25% interest compounded annually.

b) monthly payments of $375, for 7 years, at 2.99% interest compounded monthly.

2. Suppose you want to purchase a car worth $30 000 in 5 years. How much must you save monthly if your money is invested at 3.75% interest compounded monthly?

3. Rhys borrowed $20 000 to go to college at an interest rate of 5.76% compounded quarterly for 3 years.
a) What is his quarterly repayment?

b) What is the total cost of his loan?

4. Liam has a loan to buy a bedroom suite and living room set. The store offers $0 down and an interest rate of 3.99% compounded monthly for 2 years. Liam's monthly repayment is $200. What is the present value of the furniture?

5. Suppose you start regular monthly savings on your 16th birthday to have saved $1 000 000 by the time you are 65 years old. The best interest rate you can get is 3.95% compounded monthly. What is the monthly deposit?

6. Leah has a $950 000 mortgage that she wants to pay off in 20 years. The mortgage provider charges an effective interest rate of 3.6% compounded monthly.

a) What is Leah's approximate monthly payment?

b) How much interest does she pay in total?