8.1 Angles in Standard Position

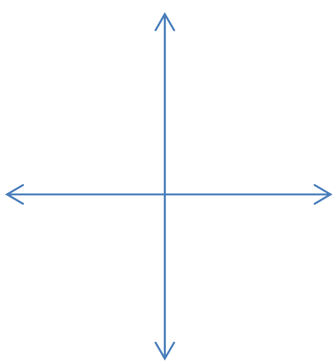
What is Trigonometry? A branch of Math that analyses and calculates the relations between the ________________ and the __________________________________ of triangles.

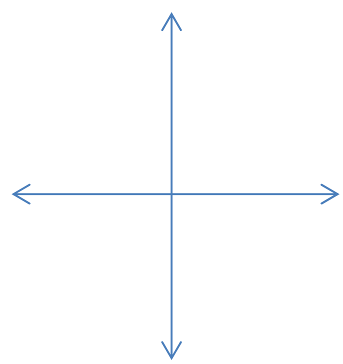
We know from grade 10 math: Sin ∠A =   Cos ∠A =   Tan ∠A =

Angles in Standard Position, $0^o \le \theta \le 360^o$

On a Cartesian plane (   ), you can generate an angle by rotating a ray about the origin. The starting position of the ray, along the positive x-axis, is the ___________________________ of the angle. The final position, after a rotation about the origin, is the __________________________ of the angle.

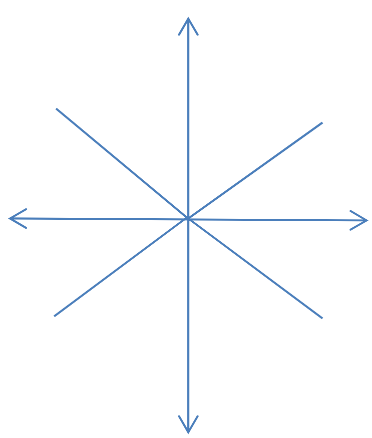
An angle is said to be an ____ if its vertex is at the origin of a coordinate grid and its initial arm coincides with the positive x-axis.





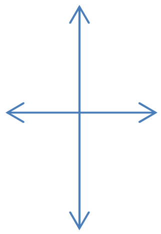
Reference Angle θ_R : the acute angle (   ) whose vertex is at the origin and whose arms are: the terminal arm of the angle and the x-axis.

In General,



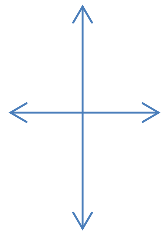
Example 1: Find the reference angle of the following principal angles:

20°



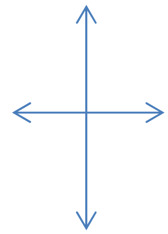
$\theta_R =$

160°



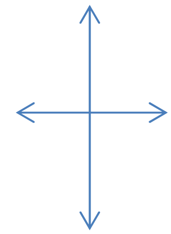
$\theta_R =$

337°



$\theta_R =$

500°

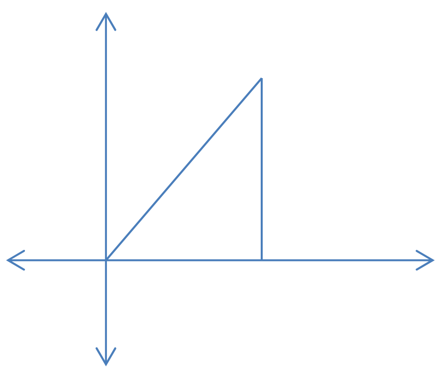


$\theta_R =$

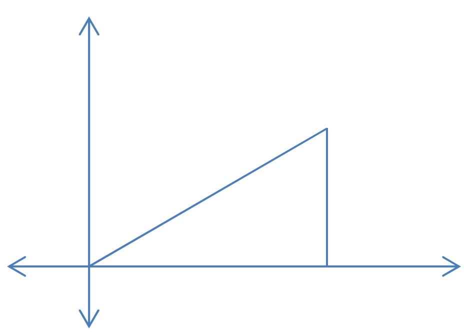
Special Angles/Special Right Triangles

For angles of $30^o, 60^o, and\ 45^o$, you can determine the ________________________________ of the trigonometric ratios: Sin, Cos and Tan.

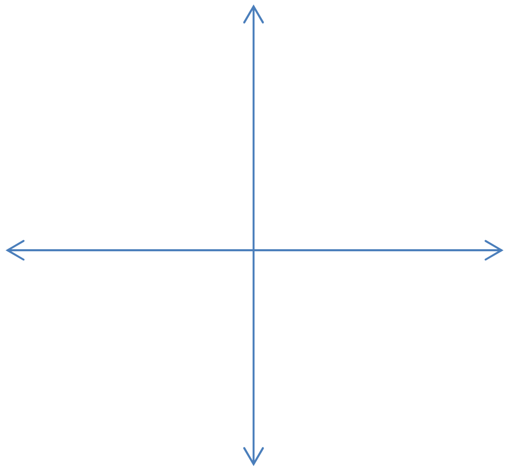
Type 1: $45^o : 45^o : 90^o$ triangle.



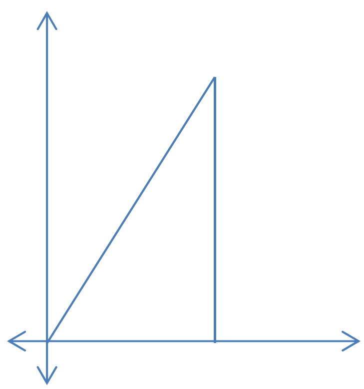
Type 2: $30^o : 60^o : 90^o$ triangle.



Example 2: Determine the **exact ratios** for $Cos\ 210^o\ \ and\ Tan\ 210^o$

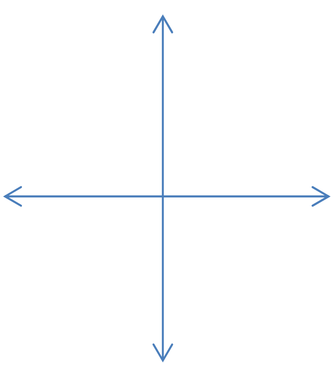


Example 3: A crane lifts its arm to an angle of 60° from the ground (horizontal). If its arm is 15m long what is the **exact value** of its vertical displacement?

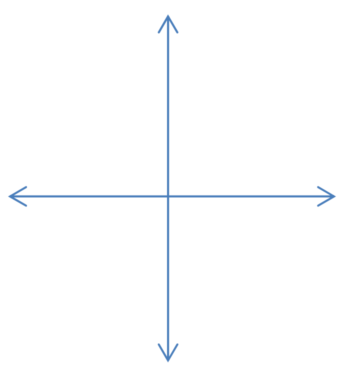


Example 4: Determine the angle in standard position when an angle of 40° is reflected

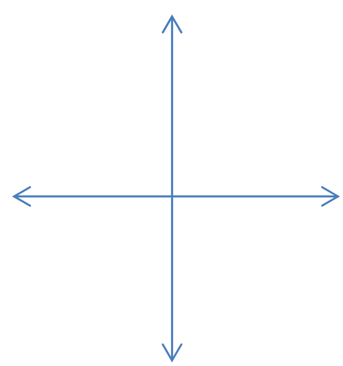
a) In the y-axis



b) In the x-axis



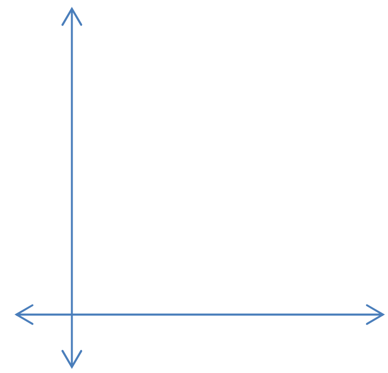
c) In the y-axis and then in the x-axis



Homework: Page 83 #2, 3abc, 4 cd, 5, 6abc, 7ab, 8, 9, 13, 15

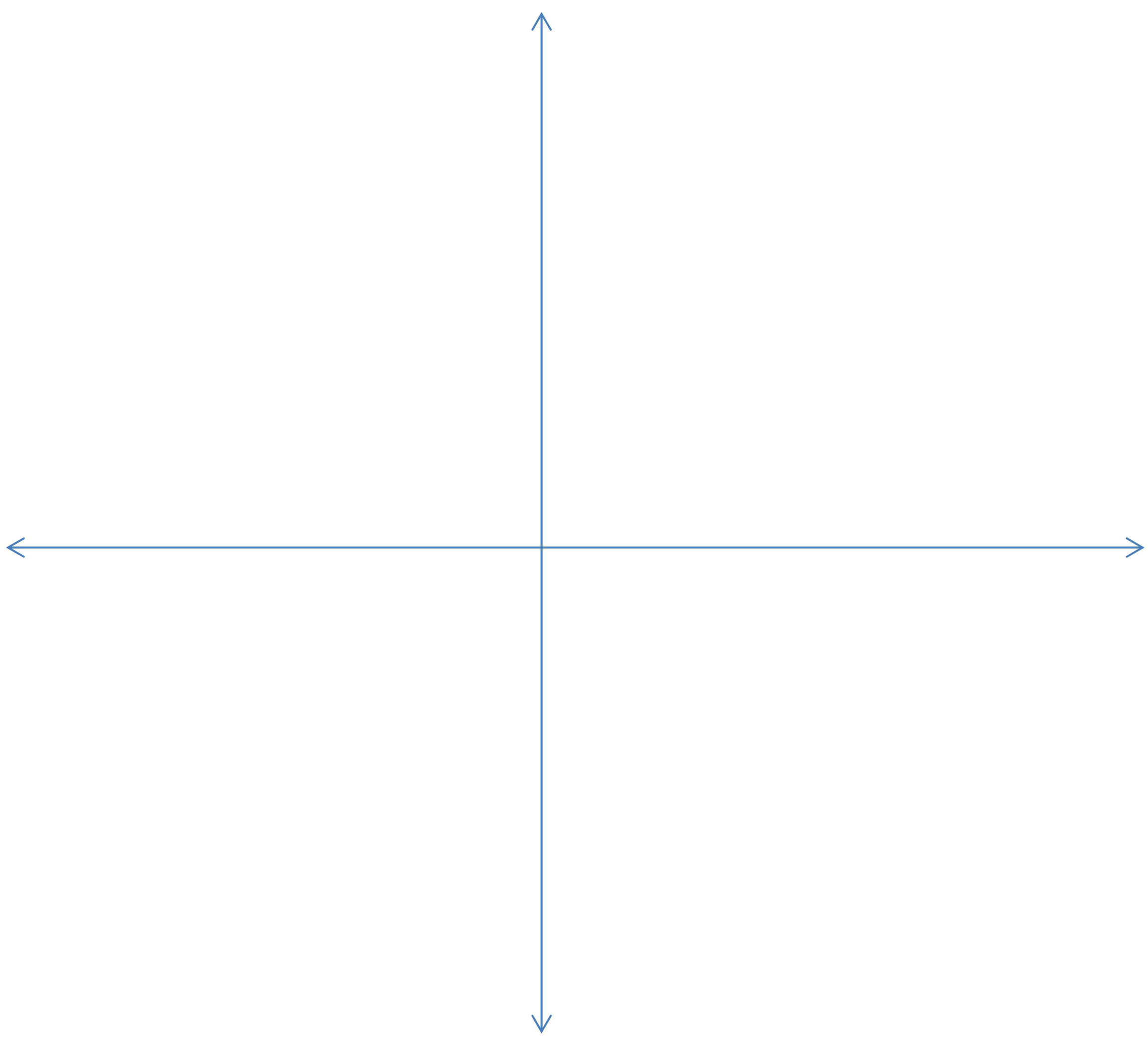
8.2 Trigonometric Ratios of Any Angle θ (part 1)

Suppose θ is any angle in standard position, and point $P\,(x, y)$ is any point on its ________________ arm, at a distance r from the origin.

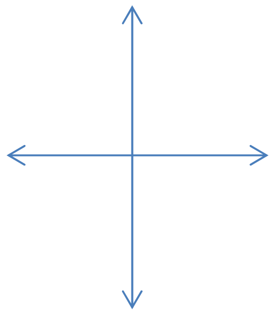


You can use a reference triangle to determine the three primary trig ratios in terms of x, y and r.

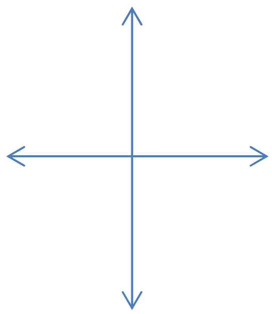
Complete the chart below to identify the signs (positive or negative) of the ratios.



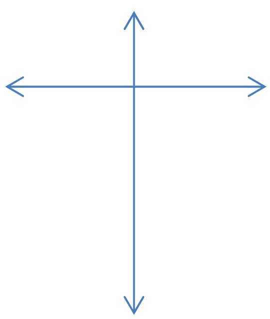
Example 1: Point P (-8, 15) lies on the terminal arm of an angle θ, in standard position. Determine the exact trig ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$.



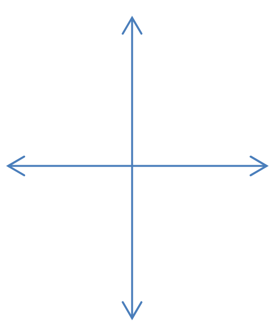
Example 2: Determine the exact value of $\cos 135^\circ$. (use special right triangle)



Example 3: Suppose θ is an angle in standard position with terminal arm in quadrant III, and $\cos\theta = -\dfrac{3}{4}$. What are the exact values of $\sin\theta$, and $\tan\theta$?



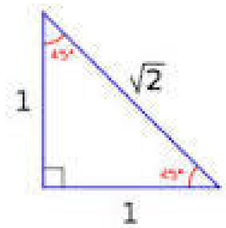
Example 4: Determine the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ when the terminal arm of quadrant angle θ coincides with the positive y-axis, $\theta = 90^\circ$.
(**quadrant angle**: an angle in standard position whose terminal arm lies on one of the axes)

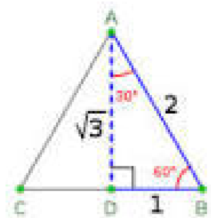


Practice: Page 96 # 2 - 7

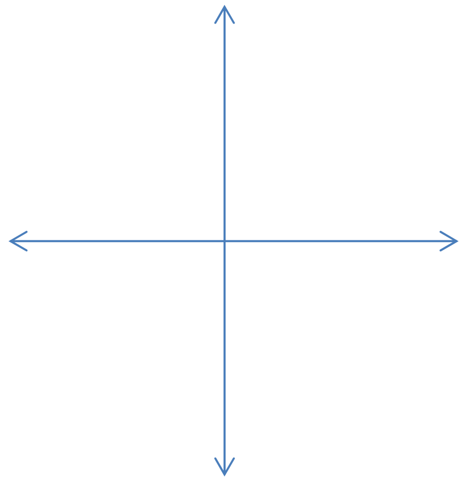
8.2 Trigonometric Ratios of Any Angle $\boldsymbol{\theta}$ (part 2)

Special Angle Triangles



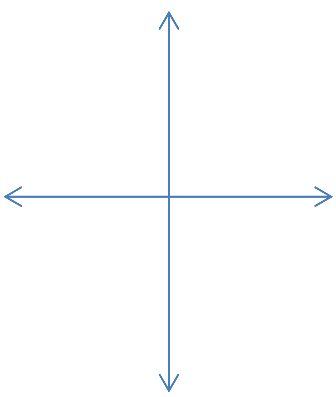


Signs of Trig. Ratios by Quadrant (C-A-S-T Rule)

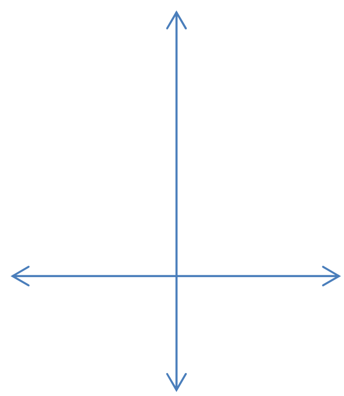


Example 1: Solve for θ (using special angle triangles, exact values)

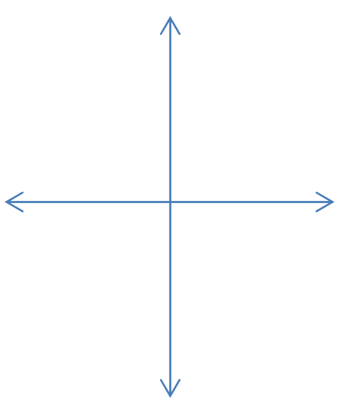
a) $\sin\theta = 0.5$, $0^\circ \le \theta < 360^\circ$



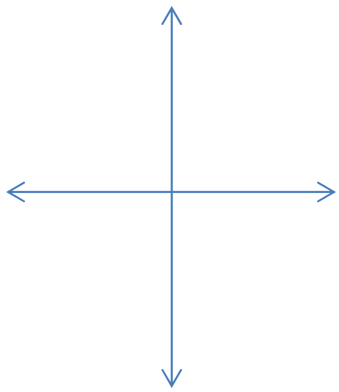
b) $\cos\theta = -\dfrac{\sqrt{3}}{2}$, $0^\circ \le \theta < 180^\circ$



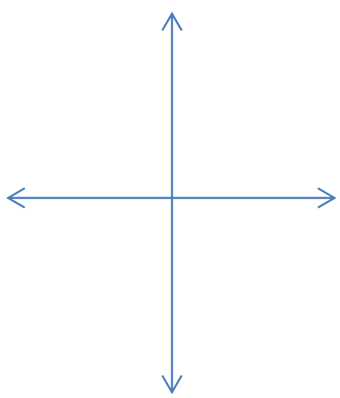
Example 2: Solve for θ $\sin\theta = -\frac{1}{\sqrt{2}}$, $0^\circ \le \theta < 360^\circ$



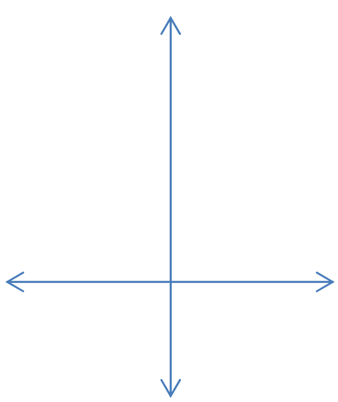
Example 3: Given $\cos\theta = -0.6753$, where $0^\circ \le \theta < 360^\circ$, determine the measure of θ, to the nearest tenth of a degree.



Example 4: Given $\sin\theta = -0.8090$, where $0^\circ \le \theta < 360^\circ$, determine the measure of θ, to the nearest tenth of a degree.



Complete the following table. Check by using your calculator.

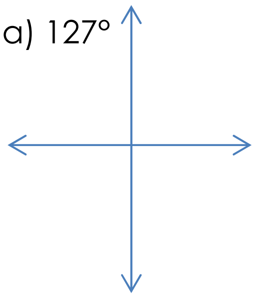


	0°	90°	180°	270°
sin		1		
Cos		0		
Tan		undefined		

8.1 & 8.2 Open Book Assignment

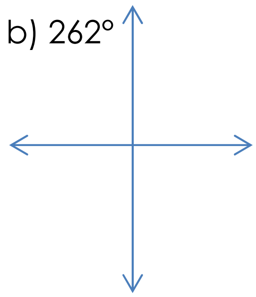
1. Find the reference angle of the following Angles in Standard Position.

a) 127°



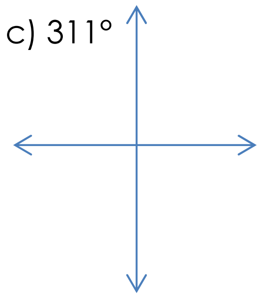
$\theta_R =$

b) 262°



$\theta_R =$

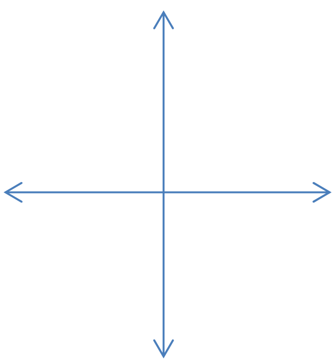
c) 311°



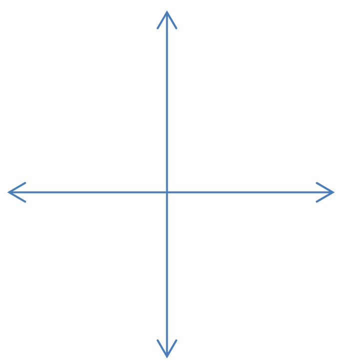
$\theta_R =$

2. Determine the Angle in Standard Position when 55° is reflected:

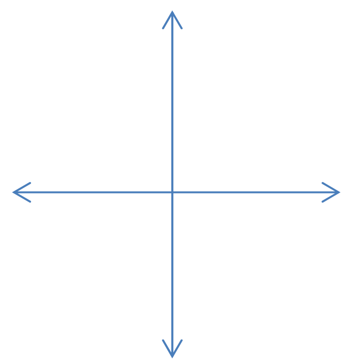
a) In the y-axis



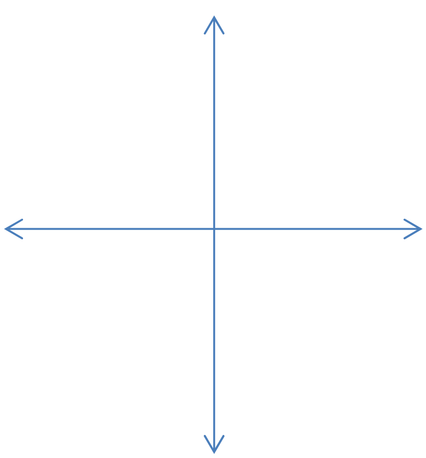
b) In the x-axis



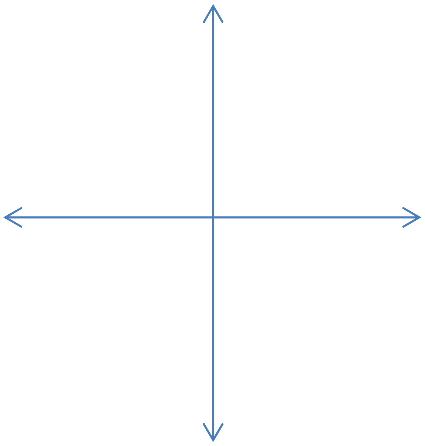
c) In the y-axis and then in the x-axis



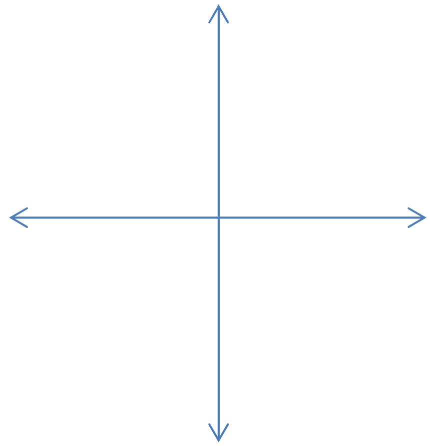
3. Point P (3 , -4) lies on the terminal arm of an angle θ , in Standard Position. Determine the EXACT trigonometric ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$.



4. Determine the EXACT value of $\sin 315^\circ$.

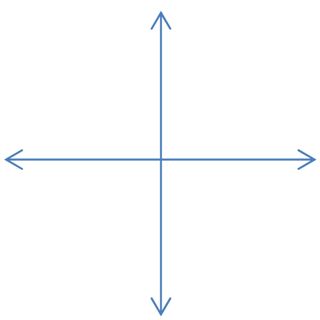


5. Determine the EXACT value of $\tan 210^\circ$.



6. Determine the EXACT value of the Sine, Cosine and Tangent values for each ratio. Show your work (x, y, r)

a) $(x, y, r) = (1, 0, 1)$

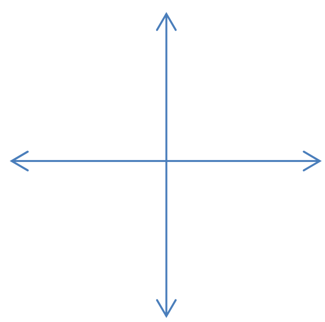


$\sin 360^\circ =$

$\cos 360^\circ =$

$\tan 360^\circ =$

b) $(x, y, r) = (0, 1, 1)$

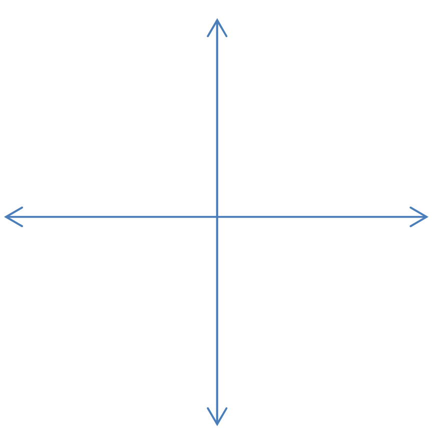


$\sin 90^\circ =$

$\cos 90^\circ =$

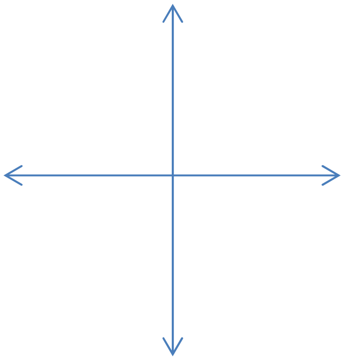
$\tan 90^\circ =$

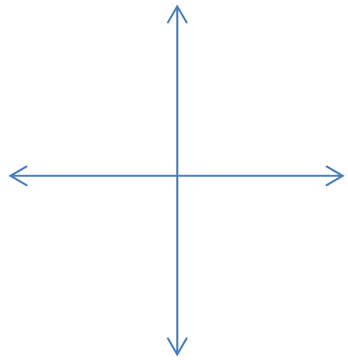
7. Suppose θ is an Angle in Standard Position with a terminal arm in Quadrant IV and $\sin\theta = \frac{-2}{5}$. What is the EXACT value of $\cos\theta$ and $\tan\theta$?



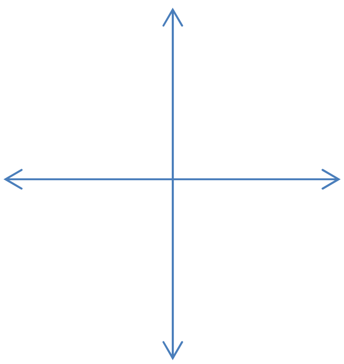
8. Solve for θ. (find the values of angle θ).

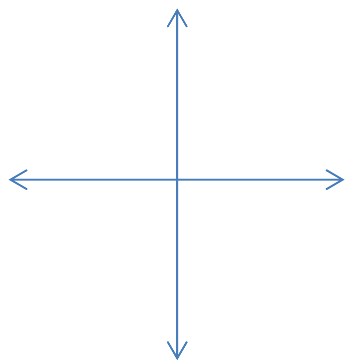
a) $\cos\theta = \frac{1}{2}$, $0^\circ \le \theta < 360^\circ$



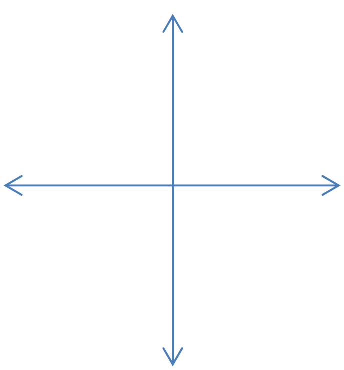


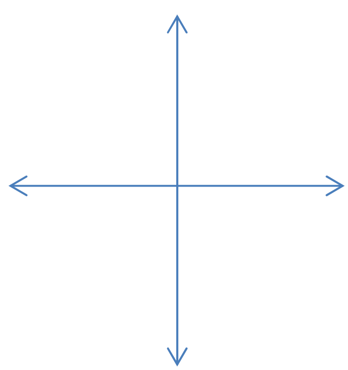
b) $\tan\theta = -1$, $0^\circ \le \theta < 360^\circ$





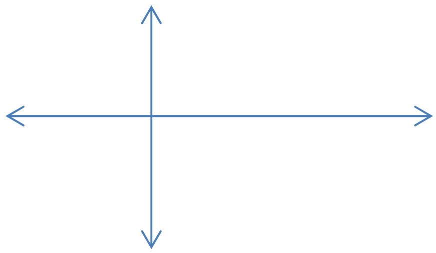
c) $\sin\theta = -\frac{\sqrt{3}}{2}$, $0^\circ \le \theta < 360^\circ$





9. Point P (7, -4) is on the terminal arm of an angle θ.

a) Sketch the angle in Standard Position.



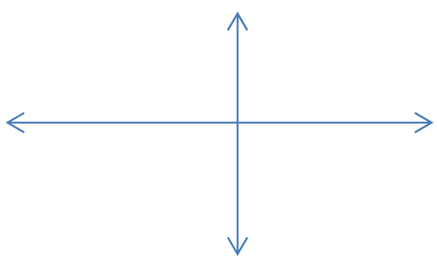
$\tan\theta =$

b) State the reference angle, θ_R, to the nearest tenth of a degree.

c) State the value of angle, θ, to the nearest tenth of a degree.

10. Point P (-4, -4) is on the terminal arm of an angle θ.

a) Sketch the angle in Standard Position.



$\tan\theta =$

b) State the reference angle, θ_R, to the nearest tenth of a degree.

c) State the value of angle, θ, to the nearest tenth of a degree.

11. Without using a calculator, state whether each ratio is positive or negative (hint: C-A-S-T)

a) $\sin 80^\circ$

b) $\tan 345^\circ$

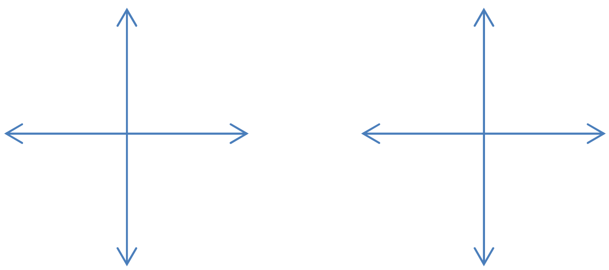
c) $\cos 181^\circ$

d) $\sin 165^\circ$

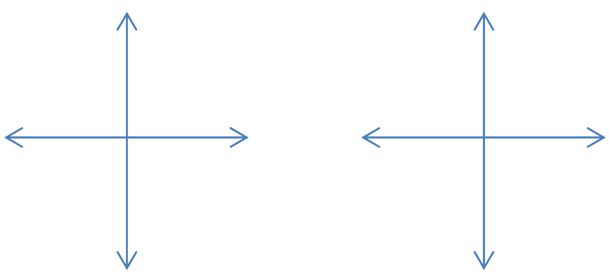
d) $\tan 280^\circ$

12. Without using technology, determine whether each statement is true or false. Prove your answer. Sketch your solution. Use exact ratios.

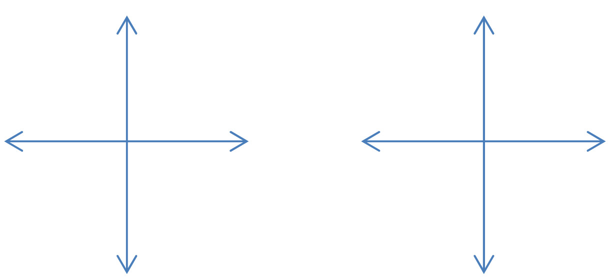
a) $\cos 135^\circ = \sin 225^\circ$



b) $\tan 135^\circ = \tan 225^\circ$



c) $\sin 60^\circ = \cos 330^\circ$



8.3 Sine Law

We know that we can use ____________________only when we have a ____________ triangle.

The **SINE LAW** is a relationship between the _____________ and the ____________ in **ANY** triangle. Let ΔABC be any triangle, where a, b and c represent the measures of the __________ opposite $\angle A$, $\angle B$ and $\angle C$ respectively.

Then, the ratios used in the **SINE LAW** are : $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

We can also use the fact that $\angle A + \angle B + \angle C = 180°$

The most simple cases of the Sine Law are _________ triangles.

Example 1: In triangle *ABC*, $\angle A = 61°$, $\angle B = 88°$ and side $c = 1.8\ km$. Find sides a and b.

Example 2: In triangle *PQR*, $\angle P = 36°$ $p = 35\ cm$ and $q = 32\ cm.$ Determine the measure of $\angle R$ to the nearest degree, and side r.

Example 3: In triangle *HIJ*, $\angle J = 115°$ $i = 4.5\ cm$ and $j = 10.8\ cm.$ Find $\angle I$ to the nearest degree.

Practice: p. 108 # 1-5

8.4 The COSINE LAW

The Cosine Law describes the relationship between the _____________ of an angle and the lengths of the three sides of any triangle.

For any ΔABC, where $a,\ b,$ and c are the ______________ of the sides opposite to $\angle A, \angle B\ and\ \angle C$ respectively, the cosine law states that:

$a^2 = b^2 + c^2 - 2bc\cos A$ or $b^2 = a^2 + c^2 - 2ac\cos B$ or _________________________________.

(Make sure you know how to solve equations properly!)

How do we know when to use **SOH CAH TOA** or **the SINE LAW** or **the COSINE LAW**?

Example 1: A surveyor needs to find the length of a swampy area near Fish Lake. She sets her transit at point A. She measures the distance to one end of the swamp as 468 m (point B) and the distance to the other side/end of the swamp as 692 m (point C). The angle of sight between the two points is 78˚. Determine the length of the swampy area, to the nearest tenth of a metre.

Example 2: The Lions' Gate Bridge in Vancouver is strengthened by triangular braces. Suppose the braces lengths are 14 m, 19 m, and 12.2 m. Determine the measure of the angle opposite the 14 metre side, to the nearest degree.

Example 3: In $\Delta ABC\ a = 11,\ b = 5,\ \angle C = 20^{\circ}$. Determine the length of the unknown side and the measures of the 2 unknown angles, to the nearest tenth.

Example 4: In $\Delta PQR\ q = 24,\ r = 18,\ \angle P = 120^{\circ}$. Solve the triangle.

Practice: P. 119 # 1 ab, 2 ab, 3 a, 4 a, 5a, 9, 10, 11, 17.

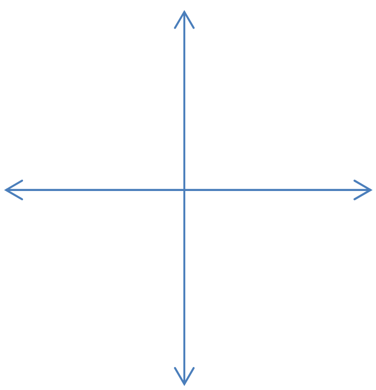
Unit 8 Review – Trigonometry

PART 1 – Angles in Standard Position

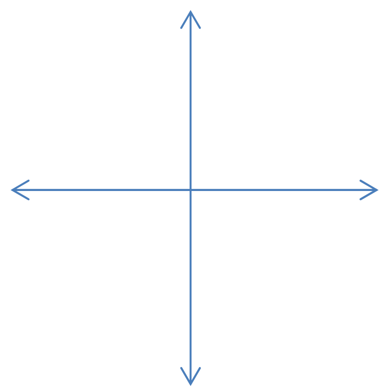
1. Draw and label your two special triangles. Label all three sides and angles.

2. Sketch the following angles in standard position and find their reference angles.

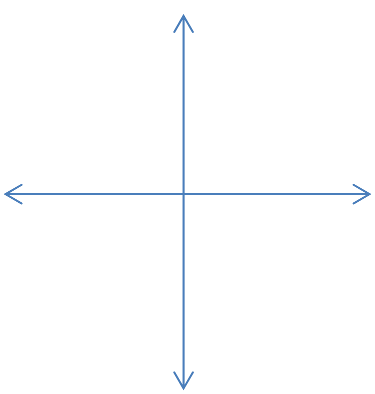
a) $\theta = 150^\circ$



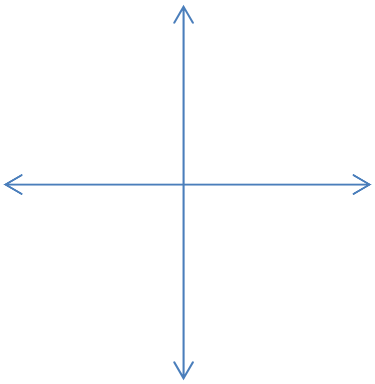
b) $\theta = 215^\circ$



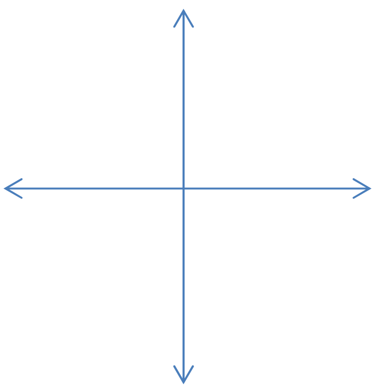
3. Determine the measure of the three other angles in standard position, $0^\circ \le \theta \le 360^\circ$, that have a reference angle of 35°.



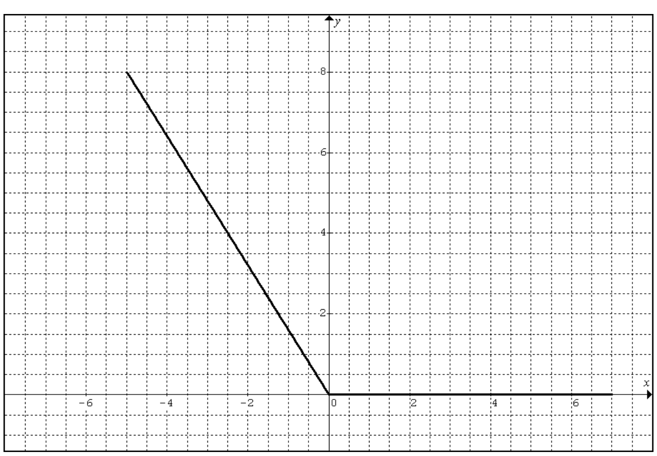
4. Point $P\ (2,-6)$ lies on the terminal arm of angle θ, in standard position. Determine the exact trig ratios for $\sin\theta,\ \cos\theta,$ and $\tan\theta$.



5. Point $P\ (-12,5)$ lies on the terminal arm of angle θ, in standard position. Determine the exact trig ratios for $\sin\theta,\ \cos\theta,$ and $\tan\theta$.

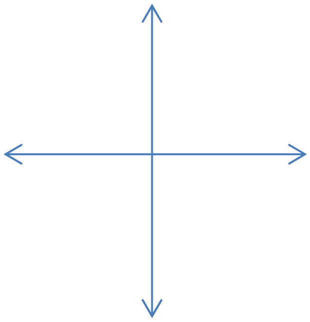


6. Angle θ is exactly 120°. Determine the exact values of the sine, cosine, and tangent ratios of this angle in standard position.

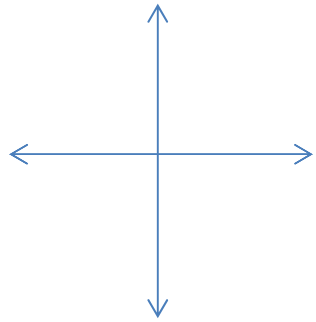


7. Determine the exact value of the following angles:

a) $\sin 150^{\circ}$

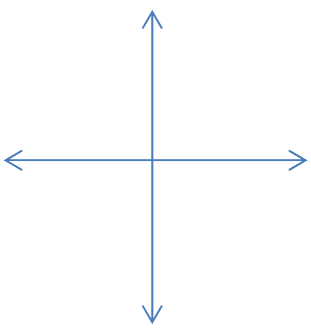


b) $\tan 315^{\circ}$

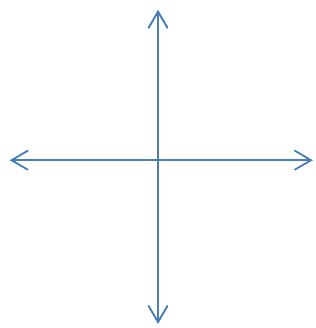


8. . Solve for θ. (Find the values of angle θ.)

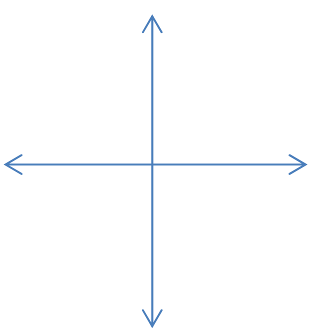
a) $\cos\theta = -\frac{\sqrt{3}}{2}$, $0^{\circ} \leq \theta < 360^{\circ}$



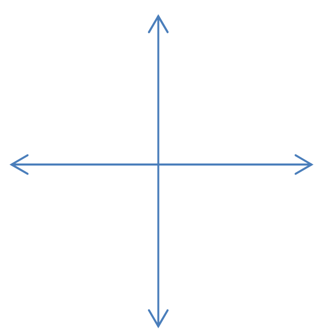
c) $\tan\theta = 1$, $0° \leq \theta < 180°$



b) $\sin\theta = -\frac{1}{2}$, $0° \leq \theta < 270°$



d) $\cos\theta = \frac{1}{2}$, $0^{\circ} \leq \theta < 360^{\circ}$



PART 2 – Sine Law

9. Find side c if, in $\Delta ABC\ \angle A = 35^\circ, \angle B = 88^\circ,\ b = 44cm$

11. Solve the triangle: $\Delta ABC\ \angle A = 39^\circ, a = 10cm,\ b = 14cm$. Round your answers to the nearest unit.

PART 3 – Cosine Law

12. In triangle PQR : $p = 17,\ q = 23,\ \text{and}\ r = 25$. Find the measure of angle Q (to the nearest degree).

13. In triangle DEF : $\angle D = 21^{\circ}$, $e = 27$, and $f = 30$. Find the measure of side d, to the nearest tenth.

More Review: p. 129 # 1- 6, 8-10, 13, 20