1. When m is an integer, describe the relationship between a^m and a^{-m}.

2. Why is there usually more than one way to determine the value of a power of the form $a^{-\frac{m}{n}}$? Use examples to justify your answer.

Exercises

A

3. Copy then complete each equation.

   a) $\dfrac{1}{5^4} = 5^\square$

   b) $\left(-\dfrac{1}{2}\right)^{-3} = (-2)^\square$

   c) $\dfrac{1}{3^\square} = 3^2$

   d) $\dfrac{1}{4^{-2}} = 4^\square$

4. Evaluate the powers in each pair without a calculator.

   a) 4^2 and 4^{-2} b) 2^4 and 2^{-4}

   c) 6^1 and 6^{-1} d) 4^3 and 4^{-3}

   Describe what is similar about the answers, and what is different.

5. Given that $2^{10} = 1024$, what is 2^{-10}?

6. Write each power with a positive exponent.

   a) 2^{-3} b) 3^{-5} c) $(-7)^{-2}$

7. Write each power with a positive exponent.

   a) $\left(\dfrac{1}{2}\right)^{-2}$ b) $\left(\dfrac{2}{3}\right)^{-3}$ c) $\left(-\dfrac{6}{5}\right)^{-4}$

8. Evaluate each power without using a calculator.

   a) 3^{-2} b) 2^{-4} c) $(-2)^{-5}$

   d) $\left(\dfrac{1}{3}\right)^{-3}$ e) $\left(-\dfrac{2}{3}\right)^{-2}$ f) $\dfrac{1}{5^{-3}}$

B

9. Evaluate each power without using a calculator.

   a) $4^{-\frac{1}{2}}$ b) $0.09^{-\frac{1}{2}}$

   c) $27^{-\frac{1}{3}}$ d) $(-64)^{-\frac{1}{3}}$

   e) $(-0.027)^{-\frac{2}{3}}$ f) $32^{-\frac{2}{5}}$

   g) $9^{-\frac{3}{2}}$ h) $0.04^{-\frac{3}{2}}$

10. Use a power with a negative exponent to write an equivalent form for each number.

    a) $\dfrac{1}{9}$ b) $\dfrac{1}{5}$ c) 4 d) -3

11. When you save money in a bank, the bank pays you *interest*. This interest is added to your investment and the resulting amount also earns interest. We say the interest *compounds*. Suppose you want an amount of $3000 in 5 years. The interest rate for the savings account is 2.5% compounded annually. The money, P dollars, you must invest now is given by the formula: $P = 3000(1.025)^{-5}$. How much must you invest now to have $3000 in 5 years?

12. Here is a student's solution for evaluating a power. Identify any errors in the solution. Write a correct solution.

$$\left(-\dfrac{64}{125}\right)^{-\frac{5}{3}} = \left(\dfrac{64}{125}\right)^{\frac{5}{3}}$$

$$= \left(\sqrt[3]{\dfrac{64}{125}}\right)^5$$

$$= \left(\dfrac{4}{5}\right)^5$$

$$= \dfrac{1024}{3125}$$

13. Evaluate each power without using a calculator.

    a) $27^{-\frac{4}{3}}$ b) $16^{-1.5}$ c) $32^{-0.4}$

    d) $\left(-\dfrac{8}{27}\right)^{-\frac{2}{3}}$ e) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$ f) $\left(\dfrac{9}{4}\right)^{-\frac{5}{2}}$

14. Michelle wants to invest enough money on January 1st to pay her nephew $150 at the end of each year for the next 10 years. The savings account pays 3.2% compounded annually. The money, P dollars, that Michelle must invest today is given by the formula

$$P = \dfrac{150\left[1 - 1.032^{-10}\right]}{0.032}.$$ How much must Michelle invest on January 1st?